JOURNAL OF AIRCRAFT
Vol. 39, No. 4, July–August 2002

Identifying Economically Optimal Flight Techniques of Transport Aircraft

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A treatment of identifying optimal flight techniques for transport aircraft with respect to direct operating cost and profit or return on investment is derived for given sector mission criteria and assumed reference time frame utilization. A series of models used to simulate maintenance and materiel costs accurately and block fuel expenditure and revenue have been introduced to force the direct operating cost and profit or return on investment expressions as continuous functions, allowing for determination of their respective minima and maxima. The selection of utilization (hourly or fixed number of sectors) per reference time frame was found to be an important precursor to what type of flight technique is to be expected. An hourly based utilization results in faster block speeds, tending toward the minimum block time threshold of a given vehicle and sector mission, whereas the fixed departures scenario yields a slower yet congruous flight technique optima requirement for direct operating cost and profit or return on investment objectives. Details are given to show how the methodology may be integrated for the purpose of conducting competitor reviews during fleet planning exercises and also how one may facilitate the optimization of conceptual aircraft designs via inspection of some useful merit parameters.

Nomenclature

a_{sls} = sonic speed at international standard atmosphere sea-level standard conditions
C_{acq} = aircraft acquisition cost and interest payable per sector (taking residual value into account becomes equivalent to depreciation cost)
C_{CONB} = block-time-related contingency cost per sector
C_{CONF} = flight-time-related contingency cost per sector
C_{crew} = crew salary cost per sector
C_{DOCS} = direct operating cost per sector and given flight technique
C^{I}_{DOCS} = explicit time-related direct operating cost component
C^{II}_{DOCS} = explicit fuel-related direct operating cost component
C^{III}_{DOCS} = fixed direct operating cost component
C_{fuel} = fuel cost for a given flight technique and sector
C_{ins} = total insurance payable per sector
C_{lease} = lease cost per sector
C_{main} = maintenance cost per sector and given flight technique
C_{mat} = materiel cost per sector and given flight technique
C_{misc} = miscellaneous indirect operating cost component per sector
C_{pax} = passenger-related indirect operating cost component per sector
C_{sales} = sales and reservation related indirect operating cost component per sector
C_{spares} = spares inventory cost per sector
C_{sund} = total sundries cost per sector
C_{TOCS} = total operating cost for given sector and flight technique
c_{acint} = aircraft acquisition cost and interest payable per reference time frame
c_{acres} = aircraft residual value per reference time frame
c_{conb} = block-time-related contingency cost per block hour
c_{conf} = flight-time-related contingency cost per flight hour
c_{crew} = crew salary cost per reference time frame
c_{grh} = ground handling charges incurred per sector
c_{hins} = hull insurance payable per reference time frame
c_{land} = landing fees incurred per sector
c_{lease} = lease cost per reference time frame
c_{main} = flight-time-dependent maintenance cost denoting theoretically most efficient work practice
$c_{main,CYC}$ = total cyclic maintenance cost per sector
$c_{main,FH}$ = total flight-time-dependent maintenance cost
$\dot{c}^{c}_{main}$ = total maintenance cost derivative with respect to block time
$\dot{c}^{p}_{main}$ = total maintenance cost derivative with respect to number of sectors completed per reference time frame utilization
c^{I}_{main} = flight-time-related maintenance cost component
$\bar{c}^{I}_{main}$ = approximate flight-time-related maintenance cost component deemed independent of segment flight time
c^{II}_{main} = fixed maintenance cost component
$\bar{c}^{II}_{main}$ = fixed maintenance cost component assuming approximate flight-time-related maintenance cost
c_{mat} = flight-time-dependent materiel cost denoting theoretically most efficient work practice
c^{I}_{mat} = flight-time-related materiel cost component
$\bar{c}^{I}_{mat}$ = approximate flight-time-related materiel cost component deemed independent of segment flight time
c^{II}_{mat} = fixed materiel cost component
$\bar{c}^{II}_{mat}$ = fixed materiel cost component assuming approximate flight-time-related materiel cost
c_{misc} = additional (direct operating) costs and fees incurred per sector
c_{nav} = navigation fees incurred per sector
c_{pins} = passenger and/or distance-related insurance rate
c_{spint} = spares inventory acquisition cost and interest payable per reference time frame

Received 16 June 2000; revision received 13 July 2001; accepted for publication 19 February 2002.

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c_{spres} = spares inventory residual value per reference time frame
$k^{\mathrm{I}}_{\mathrm{IOC}}$ = proportion of total yield that accounts for C_{pax} indirect operating cost component
$k^{\mathrm{II}}_{\mathrm{IOC}}$ = proportion of total direct operating cost that accounts for C_{misc} indirect operating cost component
$k^{\mathrm{III}}_{\mathrm{IOC}}$ = cost coefficient used for C_{sales} indirect cost component
$k^{\mathrm{IV}}_{\mathrm{IOC}}$ = fixed cost coefficient that accounts for C_{misc} indirect operating cost component
k_{main} = constant depicting fraction of maintenance cyclic to maintenance flight-time-dependent cost
k_{mat} = constant depicting fraction of materiel cyclic to materiel flight-time-dependent cost
k_1 = constant depicting the impact of higher speed technique attributes to assorted combinations of intermediate speed schedules with respect to block fuel for a given sector
k_2 = constant depicting the impact of higher speed technique attributes to assorted combinations of intermediate speed schedules with respect to block fuel for a given sector
k_3 = constant depicting the impact of slower speed technique attributes to assorted combinations of intermediate speed schedules with respect to block fuel for a given sector
k_4 = constant depicting the impact of slower speed technique attributes to assorted combinations of intermediate speed schedules with respect to block fuel for a given sector
k_5 = constant required for regression between block time (abscissa) and expended block fuel (ordinate) for a given sector
M = Mach number
N_s = number of sectors completed per reference time frame
P = profit or return on investment (denoted by prime) attributable to flying services for given sector mission and reference time frame, before income taxes, nonoperating items such as retirement of property and equipment, affiliated companies, and subsidies
P_{opt} = profit or return on investment (denoted by prime) global maximum
P_S = preoptimum profit or return on investment (denoted by prime) rise rate
P_{SEC} = profit for given sector mission and flight technique
P_{SS} = postoptimum profit or return on investment (denoted by prime) decay rate
p_f = price of fuel per unit weight
s = sector distance for given mission
s_{be} = break-even sector distance where profit or return on investment is zero
s_{dec} = reference sector distance where the postoptimum profit or return on investment (denoted by prime) decay rate is measured
s_i = initial estimate for break-even sector distance numerical scheme
s_n = upper-sector distance threshold of the surveyed sector distances
s_o = lower-sector distance threshold of the surveyed sector distances
s_{opt} = sector distance where profit or return on investment (denoted by prime) global maximum occurs
s_{ref} = reference sector distance used for yield modeling
T_u = total reference time frame utilization
t = block time for given sector and flight technique
t_a = turn-around time
t_{man} = time allowance for startup, taxi-out, and taxi-in
$t_{\mathrm{maxP\text{-}ROI}}$ = optimal profit or return on investment block time for a given sector mission
t_{mincost} = cost optimal block time for given sector mission
t_{minfuel} = block time required to complete a sector mission resulting in the lowest possible block fuel
t_{mintime} = lowest possible block time required to complete a sector mission
t_n = block time equal to the upper applicable threshold of a regressed maintenance cost model
t_o = block time equal to the lower applicable threshold of a regressed maintenance cost model
W_{fuel} = block fuel required to complete a sector mission for a given flight technique
$W_{f,\mathrm{minfuel}}$ = lowest possible block fuel required to complete a sector mission
$W_{f,\mathrm{mintime}}$ = block fuel required to complete a sector mission in the lowest possible block time
Y_{SEC} = total revenue for a given sector mission
y_1 = yield generated at a reference sector distance
y_2 = constant depicting yield variation with sector distance
y_3 = constant depicting yield variation with sector distance
α_{main} = constant coupling maintenance flight hour cost to segment flight time
α_{mat} = constant coupling materiel flight hour cost to segment flight time
β_{main} = potential regression parameter accounting for segment flight time influence on maintenance flight hour cost
β_{mat} = potential regression parameter accounting for segment flight time influence on materiel flight hour cost
θ = temperature lapse ratio
λ = passenger load factor for given sector mission
Φ_α = linear sector distance gradient coefficient in profit or return on investment (denoted by prime) response model
Φ_β = linear sector distance constant in profit or return on investment (denoted by prime) response model
Φ_δ = exponential sector distance coefficient in profit or return on investment response model
Φ_ε = coefficient representing the asymptotic behavior in the profit or return on investment (denoted by prime) response model
Φ_χ = exponential constant in profit or return on investment response model
ϖ = adjusted cost differential with respect to block time or profit differential with respect to number of sectors completed per reference time frame
ϖ' = adjusted ancillary profit differential with respect to number of sectors completed per reference time frame

Introduction

IT is becoming increasingly important for designers of transport aircraft to be well versed in how commercial airline operators establish the feasibility of introducing new equipment types for fleet planning. Airline economics now dictate the need for more flexible commercial transports, thus invalidating the traditional approach of focusing on the design point specifications and giving little regard to off-design sensitivities. One well-known example of this philosophy is the act of oversimplifying the procedural aspects of en route performance to one universally applicable Mach number or standard Mach, commonly designated as the long range cruise speed. Even though the basic requirement of operational performance is scrutinized, airlines will consider in parallel the corresponding direct operating cost (DOC), and more significantly, the profit or return on investment (P-ROI) generated. In the context of this study, the profit generated is attributable to flying services, before income taxes, nonoperating items such as retirement of property and equipment, affiliated companies, and subsidies. There are additional considerations beyond the control of aircraft designers. These are issues related to product support, fleet commonality and mix that offers the best flexibility in seating and loading, long-standing and exclusive associations with particular airframe manufacturers, and the dynamic of internal politics. Notwithstanding these other factors, the cost and profit functions mentioned are often used as a rational basis for any future acquisition exercises. In view of this, it can be concluded that operational en route performance should be optimized with respect to the primary objectives of cost and profit, and, more important, it seems logical that both of these aspects should be coupled in some manner, whereby it is possible to weigh the combined relative merits of different aircraft.

A complete mission flight profile trajectory, as depicted in Fig. 1, consists of three consecutive segments: climb, cruise, and descent. Each segment is subject to transversality conditions that are additional and that depend on the endpoint constraints of state variables[1]; thus, the entire flight must be analyzed as a global problem, wherein the links between all of the phases are considered concurrently. Unique and constant values of calibrated (or indicated) airspeed (CAS), or Mach number, for corresponding throttle setting are indicative of each phase with strategic switches in CAS/throttle affected during the flight in accordance with procedures detailed in a flight plan.

A sector mission is the operation of an aircraft from the end of initial climb to the end of descent, with both nodes corresponding to a height of 1500 ft pressure altitude. Flight time and flight fuel include allowances required for takeoff, initial climb, approach, and landing. The block time and block fuel include additional allowances for start-up, taxi-out, and taxi-in. The notion of flight and block definitions does not include any distance credit. Each sector mission analysis will have with it an associated reserve fuel that is carried to destination. Reserve fuel is a contingency allocation usually consisting of an alternate or diversion flight over a designated distance, operation in a holding pattern for a specified duration and given altitude, possibly a contingency fuel proportional to the flight fuel expended to complete the sector mission, and, where required, contingency fuel to cater for an extended flight of given duration and flight technique.

It is common practice to assign at least two distinct climb modes, or, more specifically, two different speed schedules for climb control, each consisting of a fixed CAS and Mach speed. The advantage with a faster climb speed schedule occurs for cruise fractions (ratio between cruise distance and sector distance) less than around

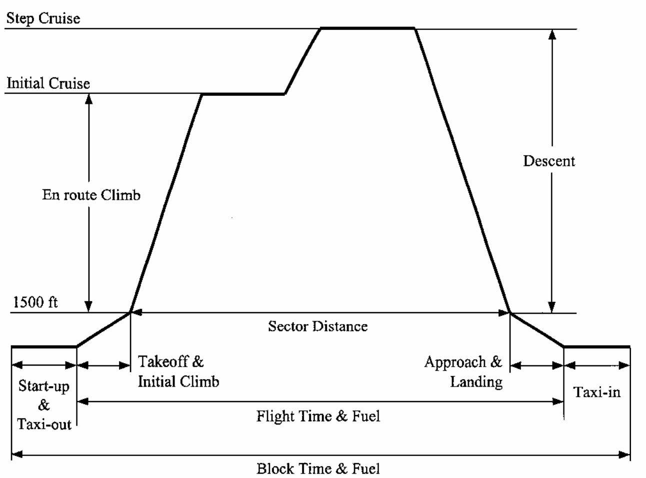

Fig. 1 Flight profile as defined by AEA.[2]

0.80 (or sector distance $\leq$1000 n mile) where possibilities in conducting further time, cost, or profit function optimization can take place.[2] To elaborate, opportunities might arise in generating an unconstrained optima that was previously constrained using a single speed schedule premise because faster climb speed schedules (CLB mode H) encourage cruise soaking, or the exchange of cruise distance for climb, which leads to block time reductions. This is especially the case for regional-type sector missions. A slower climb speed schedule (CLB mode L) enables closer adherence to fuel optimal procedures during climb, thereby enhancing range capability. In this way, CLB mode L and CLB mode H speed schedule definitions are formulated with respect to optimal climb trajectory profile state and time function adherence and designated divergence criteria, respectively. Normally, one would describe speed schedules for descent much in the same way as was mentioned for climb control. Three options are usually available: rate of descent (ROD) as a control variable, a Mach/CAS schedule as the control variable, and Mach/CAS speed schedule as the control variable with ROD used as the ancillary constraint.

Block speed (sector distance divided by block time) variation for a given reference time frame utilization (total operating or block hours for a given period of time, for example, per annum) results in markedly different speeds when optimal fuel usage (minimum fuel), optimal time expended (minimum time), minimum DOC, and maximum P-ROI are compared for fixed sector distances and mission criteria.[3] Identification of these speeds enables the formulation of optimal flight techniques or a formal definition of flight operational procedures consisting of distinct climb, cruise, and descent modes at a suitable flight level(s).

The DOC consists of three major contributors, two of which are interrelated. The first and second are designated as a flight technique source consisting of time and fuel costs, in which changes in block speed induce corresponding changes in cost of time and fuel relative to the speed increment. Moreover, the time-related cost may also be sensitive to the influence of variations in reference time frame utilization, which measures the productivity or number of sectors completed for given period of a vehicle. The third, independent of flight technique, refers to an operating cost that is not proportional to the economic value of speed or utilization, but is related to the act of completing a sector mission and, hence, is considered fixed.

Common practice among aircraft manufacturers is to compare only DOC between vehicles of varying productivity capabilities, which can, on occasions, be a questionable basis. There are instances, namely, the way in which aircraft utilization is defined, where the P-ROI objective might emphasize the importance of block speed, yielding a condition for economic optimality incongruous with minimum DOC. Fundamentally, P-ROI should be viewed as the most comprehensive of all of the objective function criteria usually considered for commercial aircraft design proposals and competitor analyses, but has been neglected in the past because of the added complexity in computing such results. Primary contributors to P-ROI include a multifaceted tradeoff between revenue and total operating cost (TOC) constrained by the influence of productivity for a given reference time frame as well as the quantity of available seat-miles completed therein.

The purpose of this paper is to derive expressions for the DOC, TOC, and P-ROI of a given aircraft and sector mission criteria and to propose a method in which it is possible to identify the associated economically optimal flight techniques. The final aim is to extend this knowledge further by offering an array of tangible merit functions related to operational performance and economics for the purpose of coupling these subspaces into the traditional conceptual aircraft design optimization process. Figure 2 offers a graphical perspective to assist in elucidating the interrelationship between vehicular attributes, operational performance, DOC, and P-ROI, and to serve as an outline for determining constituent working parameters, assumptions on which the calculations are based, as well as the flow to produce the requisite objective and merit functions. This interdependent yet concurrent process will be described in detail in the sections to follow.

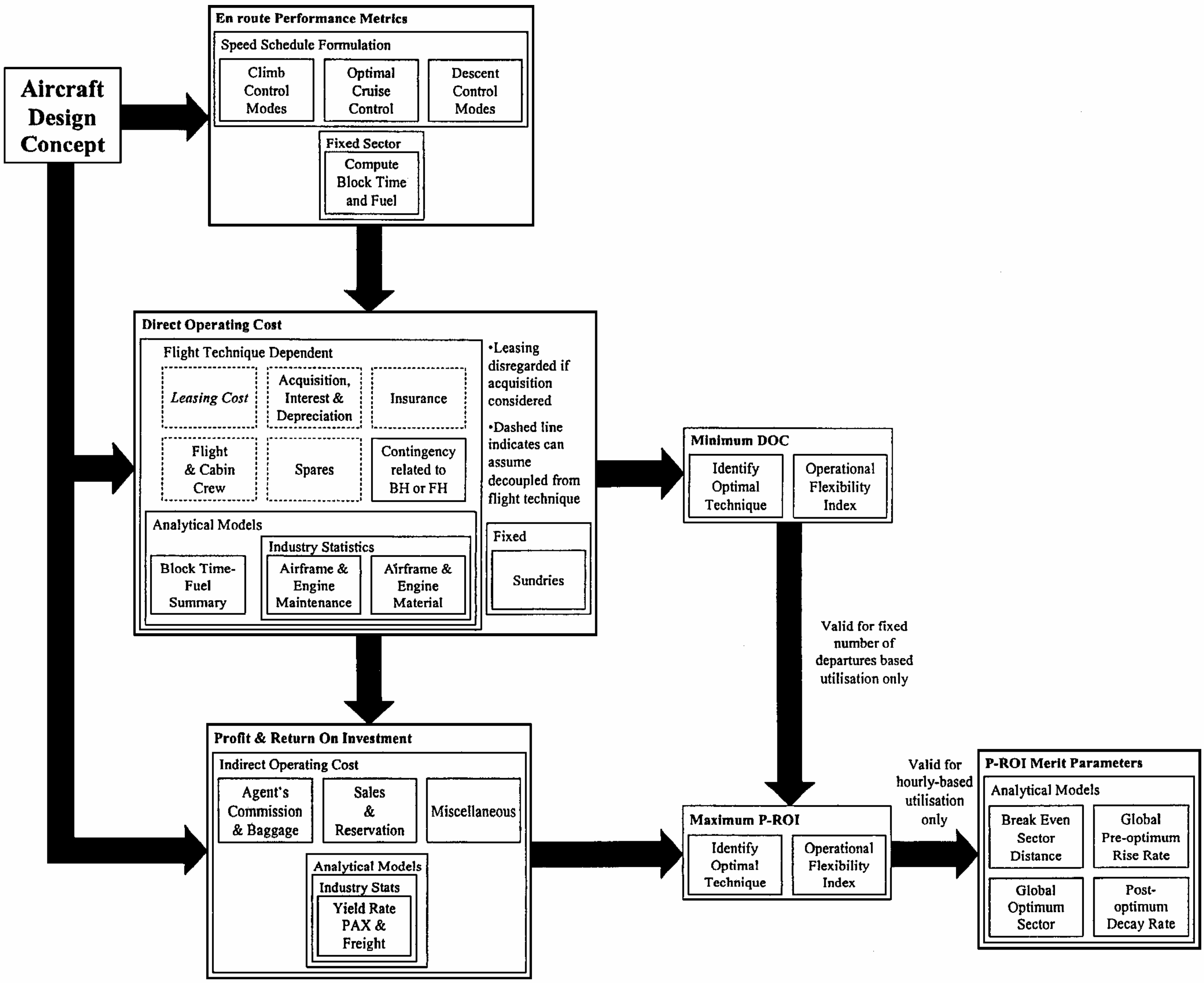

Fig. 2 DOC and P-ROI computation procedure flowchart.

Utilization for Given Reference Time Frame

The myriad of cost and revenue expressions to be presented shall become normalized to a per sector mission basis to afford a measure of effectiveness against a given flight technique. This is achieved by first allocating an assumed utilization over a period of, for example, one year, and then consequently expressing it as an equivalent number of sectors. Aircraft utilization is governed by the ratio of flight time to the ground time spent loading and unloading the vehicle, any airport restrictions on night flying, and the frequency of operations as dictated by public demand throughout the diurnal and seasonal cycles. In any traffic system, the initial planned goal is to fly the aircraft as much as possible. The number of sectors per reference time frame for a given sector mission and flight technique is simply computed by dividing the total number of operating hours of utilization by the summation of single mission block and turn-around times, thus,

$$N_S = T_u/(t + t_a) \tag{1}$$

For instances where utilization is assumed to be in block hours (BH), the turn-around time t_a is taken to be zero. Typically, industry practice is to assume the utilization or T_u of commercial aircraft to vary between 2000–4000+ BH per annum, with the lower bound of this interval akin to regional aircraft usage and the upper bound characteristic of long-range equipment. The utilization assumption is very important because it can influence both the productivity and cost attributes of airline operation. To assist in deciding an appropriate T_u, the Association of European Airlines (AEA)[2] suggest using

$$T_u = [3750/(t + t_a)]t \tag{2}$$

The declaration of T_u as presented in Eq. (2) implies that utilization is proportional to block time and, hence, is a function of flight technique. A quick sensitivity analysis shows the variation between upper and lower bounds would not exceed $\pm 10\%$, thus leading one to conclude that a philosophy of setting T_u as fixed to simplify matters is also acceptable. Otherwise, in either case, the methods of identifying economically optimal flight techniques to follow are equally applicable regardless of the nature of assumed utilization; when using Eq. (2), the only stipulation is that T_u must be computed dynamically before proceeding with the algorithm.

Another alternative is to assume a fixed number of departures, which means the parameter N_S may be expressed as a quantity independent of flight technique, and it shall be shown later that this assumption produces significantly different optima compared to an hourly based utilization.

DOC

A number of techniques for the calculation of DOC are reported in literature.[2,4–7] The flight-technique-dependent costs for a given sector mission are those including lease (if applicable), aircraft acquisition and interest due to the initial cost, aircraft and passenger insurance (consisting of both flight-technique-dependent and independent components), air crew, spares inventory, aircraft maintenance, aircraft materiel, and fuel consumption. The costs incurred independent of flight technique include navigation, landing, and handling charges. The cost components outlined here are all with respect to an hourly based reference time frame utilization assumption (i.e., different flight techniques employed for a given sector mission result in variations of block time) and, hence, the number of sectors achievable corresponding to cost variations per sector flown. In contrast, a fixed departures utilization assumption will modify the basis for account of the time-dependent cost constituents. This aspect is to be discussed after the hourly based utilization optimal flight technique scheme has been derived.

Flight-Technique-Dependent Costs

These are costs related to aircraft specific operational performance attributes and mission requirements. This section intentionally includes aircraft ownership-related costs as constituents that can be coupled to flight technique; however, it is highlighted that many DOC studies produced by airframe manufacturers[7–9] work off the premise of cash DOCs or costs not related to aircraft ownership.

Aircraft Lease Cost

A contractual agreement by which the owner of the vehicle allows another party to use it for a specified time in return for a settled hire rate. Normalizing this cost from reference time frame to a sector basis produces

$$C_{\text{lease}} = (c_{\text{lease}}/T_u)(t + t_a) \tag{3}$$

where c_{lease} is expressed as currency units per reference time frame (CU/RTF) and C_{lease} as currency units per sector (CU/SECT). Note that the byproduct of leasing means that concepts like acquisition and interest payable for a depreciation period are neglected for ensuing DOC calculations.

Aircraft Acquisition, Interest Cost, and Depreciation

This cost relates to the initial capital outlay and repayment of the interest invested for aircraft procurement. Depreciation is the allocation of the aircraft initial cost over the operating life of the aircraft. The total aircraft acquisition cost and interest payable (taking residual value into account becomes equivalent to depreciation cost) per sector (CU/SECT) is expressed as

$$C_{\text{acq}} = [(c_{\text{acint}} - c_{\text{acres}})/T_u](t + t_a) \tag{4}$$

Aircraft and Passenger Insurance Cost

During its operational life, the aircraft is to be insured. This is commonly known as hull insurance. A supplementary cost associated with the insurance of passengers is a function of both the number of passengers (PAX) and/or the distance covered by the aircraft. For an assumed passenger load factor and sector distance, this contribution becomes

$$C_{\text{ins}} = (c_{\text{hins}}/T_u)(t + t_a) + c_{\text{pins}}\lambda\,\text{PAX}s \tag{5}$$

where c_{hins} has units of CU/RTF, c_{pins} is in currency units per available seat-mile (CU/ASM), and C_{ins} is in CU/SECT.

Crew Cost

The crew cost includes salary of the pilots and the cabin crew. If c_{crew} is defined in CU/RTF

$$C_{\text{crew}} = (c_{\text{crew}}/T_u)(t + t_a) \tag{6}$$

C_{crew} is then expressed in CU/SECT.

Aircraft Spares Inventory

Spares ownership involves initial investment with an added burden of interest payable on the capital for procurement, as well as allocation of the initial cost over the operating life of the vehicle. The spares allowance is usually assumed as being some percentage of aircraft purchase price with adjustments made for interest and residual value. If the total spares inventory acquisition cost and interest payable (CU/RTF) and the residual value of the spares inventory (CU/RTF) are considered concurrently, the total spares inventory cost per sector (CU/SECT) is

$$C_{\text{spares}} = [(c_{\text{spint}} - c_{\text{spres}})/T_u](t + t_a) \tag{7}$$

Contingency Costs Related to Flight Technique

Additional cost sources that have a direct coupling to block and flight time can be accounted for under the guise of contingency. For example, the cost of oil consumption may be introduced via this parameter by adjusting the volumetric cost with the volumetric requirement per block or flight hour and, thence, the total cost per hour. Other instances where costs are gauged on an hourly basis may be employed here. One typical example occurs when crew wages and penalty rates instead of fixed salaries are applicable.

A BH-dependent cost is simply

$$C_{\text{CONB}} = c_{\text{conb}}t \tag{8}$$

where the contingency cost per block hour (CU/BH) is normalized into a contingency cost per sector (CU/SECT).

Correspondingly, flight-hour-dependent costs become

$$C_{\text{CONF}} = c_{\text{conf}}(t - t_{\text{man}}) \tag{9}$$

where the contingency cost per sector CU/SECT is the product of contingency cost per flight hour (CU/FH) and the difference between block time and allowances for startup, taxi-out, and taxi-in.

Aircraft Maintenance Cost

It has been demonstrated that the maintenance cost consists of time-dependent and cyclic components.[2,6,8,9] A survey completed by Boeing Commercial Airplanes[8] provides one with an insight into the relative sensitivity of constituent aircraft systems cost to time-dependent and cyclic airframe maintenance cost components. Maintenance cost for systems that encompass air conditioning, autoflight, communications, electrical power, flight controls, fuel, hydraulic power, instruments, lights, navigation, oxygen, nacelles and pylons, and windows were found dominated by time dependency. In contrast, equally split time-cyclic dependency and predominately cyclic maintenance cost constituents were associated with systems covering equipment and furnishings, ice and rain protection, landing gear, pneumatics, water and waste, auxiliary power unit (APU), doors, fuselage, stabilizers, and wings. One generally accepted approach involves the correlation of maintenance cost to average segment flight time for given sector distance; the flight hour cost should then be some function of flight time for a given mission, whereas the associated cyclic cost should be considered as some proportion of the flight hour cost.[8] This deduction is based on the premise that influences of skill level, shop efficiency, and learning curve would impart a significant contribution to both the time-dependent and cyclic costs.[9] Figures 3 and 4 demonstrate this notion with relative cost for regional, narrow-body, and wide-body aircraft.

An all-purpose model for flight-hour-related costs in the closed flight time interval $[t_o, t_n]$ is here proposed as

$$c_{\text{main,FH}} = c_{\text{main}} + \alpha_{\text{main}} \big/ (t - t_{\text{man}})^{\beta_{\text{main}}} \tag{10}$$

and associated cyclic costs as

$$c_{\text{main,CYC}} = k_{\text{main}} c_{\text{main,FH}} \tag{11}$$

where the total maintenance time-dependent cost component (CU/FH) in Eq. (10) consists of c_{main}, the portion of the cost that is flight-time-dependent and, theoretically, the most efficient work

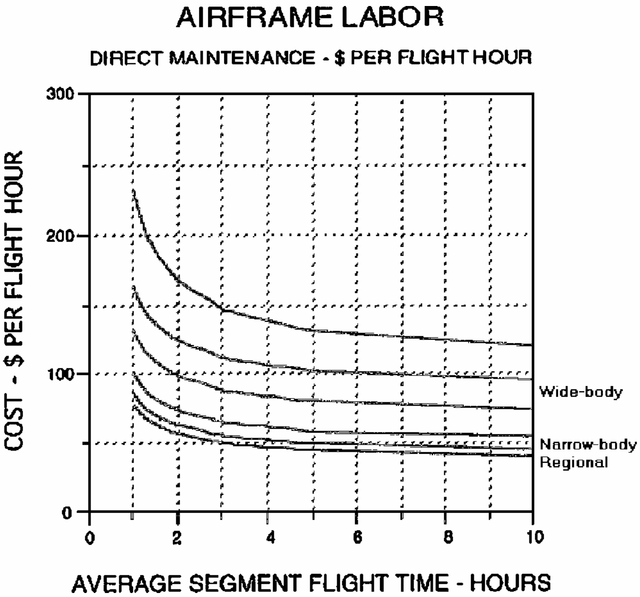

Fig. 3 Variation of airframe time-related maintenance cost with average segment flight time for various aircraft categories.[9]

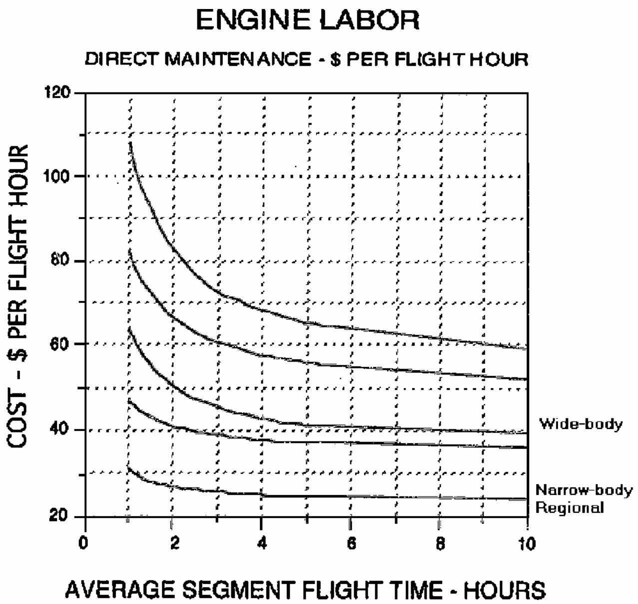

Fig. 4 Variation of propulsion time-related maintenance cost with average segment flight time for various aircraft categories.[9]

practice or learning curve asymptote; α_{main}, a constant coupling the influence of segment flight time to the flight hour cost; and β_{main}, a potential regression fit. The cyclic maintenance cost (CU/FH) given by Eq. (11) is assumed to be proportional to the maintenance time-dependent cost via k_{main}, a constant depicting the fraction of cyclic to time-dependent costs.

The parameters within the maintenance cost rate expressions given must be adjusted for changes in price level against the base statistical survey. This means the differences between nominal and current direct labor rates for a supposed burden, as well as the influence of inflation, should already be taken into account. The cyclic maintenance cost has been assumed to be some proportion of the total maintenance flight hour cost, which may not be convenient in some cases. Instances where the cyclic component is considered to be a fixed quantity regardless of flight technique may be classified under sundries. The procedure basically requires a definition that $k_{\text{main}} = 0$ and the subsequent cost be entered as a miscellaneous source.

It will be necessary to manipulate the combined influences of Eqs. (10) and (11) algebraically into a form more conducive for ease of differentiation with respect to block time. It can be demonstrated[3] that the total maintenance once coupling between flight hour cost and average segment flight time are established in conjunction with the cyclic constituent, can be alternatively expressed as

$$C_{\text{main}} = c^{\text{I}}_{\text{main}} t + c^{\text{II}}_{\text{main}} \tag{12}$$

where

$$c^{\text{I}}_{\text{main}} = \left(c_{\text{main}} + \frac{\alpha_{\text{main}}}{t(t - t_{\text{man}})^{(\beta_{\text{main}} - 1)}} \right)(1 + k_{\text{main}}) \tag{13}$$

$$c^{\text{II}}_{\text{main}} = -c_{\text{main}}(1 + k_{\text{main}}) t_{\text{man}} \tag{14}$$

When it is recognized that maintenance consists of individual airframe and propulsion contributions, the total cost can be tallied

$$C_{\text{main}} = \left(c^{\text{I}}_{a\,\text{main}} + c^{\text{I}}_{p\,\text{main}} \right) t + c^{\text{II}}_{a\,\text{main}} + c^{\text{II}}_{p\,\text{main}} \tag{15}$$

where the subscripts a and p denote airframe and propulsion components, respectively, and C_{main} is now the total maintenance cost per sector (CU/SECT).

The propulsion maintenance cost can be manipulated to reflect a variation in takeoff thrust policies together with any alterations made to en route maximum climb and cruise thrust ratings. Generally, the influence of thrust rating would be built into the model attributes of Eq. (10) from actual cost data simulating the particular configuration. In addition, investigations have demonstrated that the influence of airplane cruise speed is minimal with respect to propulsion maintenance costs.[8] In fact, the flight-time-dependent engine component overhaul is theoretically less expensive when the aircraft is operated at faster speeds. Concurrently, this cost rationalization is offset by virtue of operating at a higher thrust level, hence making redundant any consideration of throttle on cost. In those occasions where the effect of thrust rating or throttle must be considered, the k_{main} constant can be adjusted accordingly, thereby simulating this sensitivity from a modeled baseline.

Note that the cost levels used in such analysis should represent mature (stabilized) airframe and engine maintenance.[2,7] The moment a new aircraft is placed into operation, the airframe and engine maintenance costs increase at asynchronous rates from an initial low level, reach a common plateau of maturity after five to seven years of operation, and revert back to a steady increase, albeit at a less pronounced rate, due to effects imparted by age. This is a key assumption because data show the maturity factor between airframe and engine converges during this interval,[7] thereby giving scope for simplification. The total cost estimate given by Eq. (15) requires a detailed array of reliable statistical correlation. One may resort to an approximate expression[10–15] under the proviso that apt estimates of $c_{a\,\text{main,FH}} = \overline{c_{a\,\text{main}}}$ and $c_{p\,\text{main,FH}} = \overline{c_{p\,\text{main}}}$ are substituted for Eq. (10):

$$C_{\text{main}} = \left(\bar{c}^{\text{I}}_{a\,\text{main}} + \bar{c}^{\text{I}}_{p\,\text{main}} \right) t + \bar{c}^{\text{II}}_{a\,\text{main}} + \bar{c}^{\text{II}}_{p\,\text{main}} \tag{16}$$

Aircraft Materiel Cost

The expression for total materiel costs can be derived a priori based on the conclusions drawn in the maintenance cost model and a premise that both maintenance and materiel costs may be combined in the one expression.[2,6,8,9] By the use of the rationale given for Eqs. (10) and (11), and by the rearrangement of the collective influence into a form suitable for differentiation, the general model for time-related costs in the closed interval $[t_o, t_n]$ is proposed as

$$c^{\text{I}}_{\text{mat}} = \left(c_{\text{mat}} + \frac{\alpha_{\text{mat}}}{t(t - t_{\text{man}})^{(\beta_{\text{mat}} - 1)}} \right)(1 + k_{\text{mat}}) \tag{17}$$

and the cyclic contributor also becomes

$$c^{\text{II}}_{\text{mat}} = -c_{\text{mat}}(1 + k_{\text{mat}}) t_{\text{man}} \tag{18}$$

which is similar to the form of Eqs. (13) and (14). As with Eq. (15), both combine to produce the total materiel cost per sector

$$C_{\mathrm{mat}} = \left(c^{\mathrm{I}}_{a\,\mathrm{mat}} + c^{\mathrm{I}}_{p\,\mathrm{mat}}\right)t + c^{\mathrm{II}}_{a\,\mathrm{mat}} + c^{\mathrm{II}}_{p\,\mathrm{mat}} \tag{19}$$

Again, the subscripts a and p denote airframe and propulsion components, respectively. This total cost estimate will also require a detailed array of reliable statistical correlation, however, one may resort to an approximation[10–15] under the proviso that apt estimates of $c_{a\,\mathrm{mat,FH}} = \overline{c_{a\,\mathrm{mat}}}$ and $c_{p\,\mathrm{mat,FH}} = \overline{c_{p\,\mathrm{mat}}}$ are used:

$$C_{\mathrm{mat}} = \left(\bar{c}^{\mathrm{I}}_{a\,\mathrm{mat}} + \bar{c}^{\mathrm{I}}_{p\,\mathrm{mat}}\right)t + \bar{c}^{\mathrm{II}}_{a\,\mathrm{mat}} + \bar{c}^{\mathrm{II}}_{p\,\mathrm{mat}} \tag{20}$$

This cost has been intentionally separated from the total direct maintenance so that facility is given for instances where cost between maintenance and materiel are deemed mutually exclusive. A common assumption is to consider spares allowances as a fixed proportion of aircraft price[2,13–15]; this contingency is offered under the aircraft spares inventory classification.

Fuel Cost

As was discussed earlier, a complete mission trajectory is subject to transversality constraints that are additional and that depend on endpoint constraints. This means that for small enough cruise fractions, the influence of climb and descent may have a significant impact toward block fuel compared to that of cruise alone. The first step in estimating the total fuel cost for a given sector distance is to formulate a block time–fuel summary. These curves are derived from various combinations of speed schedules and flight trajectories, thus encompassing techniques for minimum time, minimum fuel, and intermediate schedules of height–energy–block fuel minima for fixed block times between these two extremes. Figure 5 shows a generic interpretation of the typical block time–fuel summary.

Because the block time–fuel summary is made up of a collection of different flight techniques, that is, combinations of distinct climb, cruise, and descent modes at specific flight level(s), the curve geometry is constructed through a combination of quasi-discrete and discrete points. The quasi-discrete portion of the curve is usually generated by a sole flight technique, commonly of highest speed schedule for climb, cruise and descent, in which flight level varies from the optimum altitude [unconstrained specific air range maximum (SAR)] or service ceiling (constrained SAR maximum) to lower altitudes until the minimum time threshold is reached. The discrete points usually consist of intermediate to low climb and descent modes combined with intermediate to long-range cruise (LRC) and maximum-range cruise (MRC) speeds at optimal altitude or service ceiling. In addition, note that, under the assumption that the margin to buffet is not violated, instances might arise where the en route specific excess power is sufficient enough to employ step cruise procedures. This aspect of performance is very difficult to predict with simplified expressions coupled to a general set of aircraft parameters and so, as a consequence, is reliant on batch calculations and

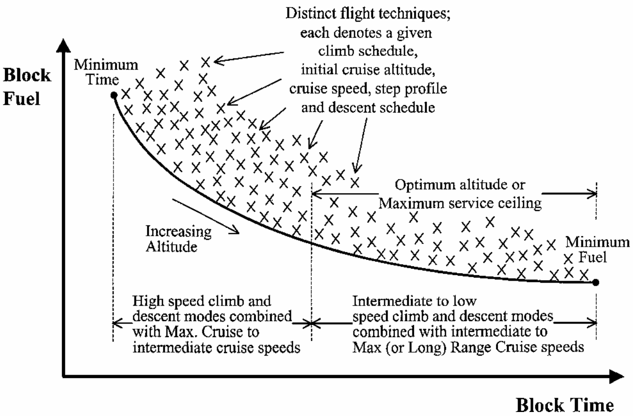

Fig. 5 Typical block time–fuel summary for a given sector distance and mission.

comparison until a collection of points describing a distinct lower boundary is established.

It is evident that the block time–fuel summary is rather complex and can not be easily represented by an analytical expression that produces a continuous function with respect to block time. The failure of this option implies that another philosophy may be required to achieve the task. A hyperbolic function appears well suited to the curve definition exercise, and a suggested model in the closed block time interval $[t_{\mathrm{mintime}}, t_{\mathrm{minfuel}}]$ is presented here as

$$\begin{aligned} W_{\mathrm{fuel}} &= W_{f,\mathrm{mintime}}(1 - k_1)\tanh[k_2(t_{\mathrm{mintime}} - t)] \\ &\quad - W_{f,\mathrm{minfuel}}(1 - k_3)\tanh[k_4(t_{\mathrm{minfuel}} - t)] + k_5 \end{aligned} \tag{21}$$

where $W_{f,\mathrm{mintime}}$ is the block fuel for a minimum time flight technique, k_1 and k_2 are constants that allow for the impact of different higher speed technique attributes to assorted combinations of intermediate schedules, $W_{f,\mathrm{minfuel}}$ is the block fuel for a minimum fuel flight technique, k_3 and k_4 are constants that allow for the impact of different slower speed technique attributes to assorted combinations of intermediate schedules, and k_5 is a constant. Because the k properties are intended to represent vehicular en route performance attributes related to aerodynamic and propulsion characteristics, extensive investigations were conducted to ascertain if expressions could be developed to quantify their respective magnitudes. Results hitherto indicate these coefficients cannot easily be related to a specialized set of design parameters or even expressed as consistent continuous functions of variables such as, for example, sector distance. This unfortunate circumstance is attributable to the complex nature associated with block time–fuel curve creation; therefore, the only recourse is to model the collectivized interdisciplinary result and weigh the relative sector mission merits of one complete aircraft against another.

When p_f is defined as the price of fuel per unit weight

$$\begin{aligned} C_{\mathrm{fuel}} &= p_f\{W_{f,\mathrm{mintime}}(1 - k_1)\tanh[k_2(t_{\mathrm{mintime}} - t)] \\ &\quad - W_{f,\mathrm{minfuel}}(1 - k_3)\tanh[k_4(t_{\mathrm{minfuel}} - t)] + k_5\} \end{aligned} \tag{22}$$

the total fuel cost per sector (CU/SECT) can be calculated.

Sundries

Sundries entail costs not exclusively related to flight operational characteristics, that is, those parameters that are not strictly functions of block time. This can consist of landing fees, navigational, and ground handling charges, which, incidentally, vary from country to country. These costs are primarily related to aircraft gross weight, sector distance, and payload complement and, thus, can be considered constant for fixed sector distances. Other costs having no direct coupling to time and not addressed here may then be categorized as miscellaneous costs. The aforementioned contributors to the total sundries cost collectively are summed as

$$C_{\mathrm{sund}} = c_{\mathrm{land}} + c_{\mathrm{nav}} + c_{\mathrm{grh}} + c_{\mathrm{misc}} \tag{23}$$

DOC for a Given Sector Mission

On summation of the aforementioned cost constituents related to flight operational aircraft, as well as sector specific aspects, the total DOC for a given sector mission and flight technique becomes

$$\begin{aligned} C_{\mathrm{DOCS}} &= [C_{\mathrm{lease}}] + C_{\mathrm{acq}} + C_{\mathrm{ins}} + C_{\mathrm{crew}} + C_{\mathrm{spares}} + C_{\mathrm{CONB}} \\ &\quad + C_{\mathrm{CONF}} + C_{\mathrm{main}} + C_{\mathrm{mat}} + C_{\mathrm{fuel}} + C_{\mathrm{sund}} \end{aligned} \tag{24}$$

By the substitution of the array of itemized cost constituents presented earlier into Eq. (24), the total DOC, assuming an hourly based reference time frame utilization, becomes quite convoluted. To further analyze an expanded form of Eq. (24) in a coherent manner, a more palatable structure should be developed. An option is to partition C_{DOCS} into time-dependent, fuel-dependent (which is also a function of time), and ancillary parts

$$C_{\mathrm{DOCS}} = C^{\mathrm{I}}_{\mathrm{DOCS}} + C^{\mathrm{II}}_{\mathrm{DOCS}} + C^{\mathrm{III}}_{\mathrm{DOCS}} \tag{25}$$

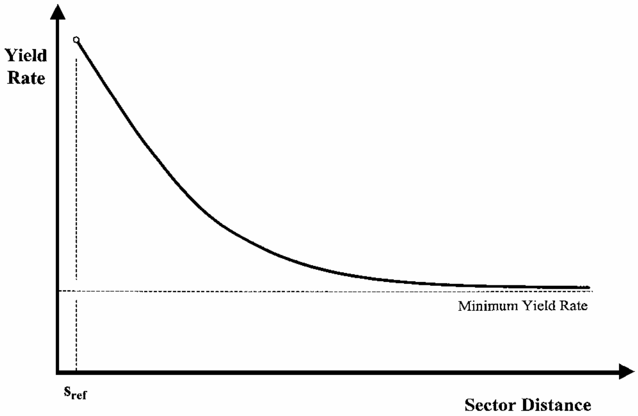

Fig. 6 Variation of yield rate with sector distance for model formulation purposes.

One practice is to evaluate the relative merits of a given aircraft design by assuming a fixed number of departures or sector missions per reference time frame. The implication is that utilization-dependent parameters such as total ownership and crew salary are no longer coupled to variations in flight technique or block time. An assumed reference time frame utilization expressed in hours has the inherent characteristic of continually varying the number of possible sectors completed per reference time frame with flight technique or block time. An identification of minimum DOC chiefly involves maximizing the number of sectors completed per reference time frame, thus emphasizing higher-speed flight techniques. In this instance, the fixed departures assumption has a weaker criterion for maximizing the number of sectors; the proportion of time-dependent cost is less significant compared to the fuel cost and, thus, is expected to result in slower cost optimal block speeds.

Yield Rate and Revenue

The yield rate is an indicator of the market in which the airline operates. This quantity is a measure of ticketing prices assumed as some function of available seat-miles. One salient characteristic any representation of yield must have is the recognition that there is a tendency toward an asymptotic value for longer sector distances (Fig. 6), thus reflecting the reducing trend for the operator cost per available seat-mile,[6,7,13–15] or an appreciation of unit costs for short-haul operations that are higher than those of longer-range flights.

The total income generated for a given flight is defined as the revenue. For an assumed passenger load factor and sector distance, on formulation of a feasible yield rate model, the total revenue per flight becomes

$$Y_{\mathrm{SEC}} = y_1 \lambda \,\mathrm{PAX} s\{1 + y_2 \tanh[y_3(s_{\mathrm{ref}} - s)]\} \tag{26}$$

where y_1 is the yield at the reference sector distance in CU/ASM, y_2 (dimensionless), and y_3 (per nautical mile) are constants depicting the impact of longer sector distances to the yield rate, and s_{ref} is the reference sector distance (nautical miles). As a supplement to the treatment given, the commercial transportation of scheduled and nonscheduled freight adheres to a similar edict; the yield rate would still be modeled using Eq. (26), but expressed in currency units per load ton-mile instead of available seat-miles.

Indirect Operating Cost

The indirect operating costs (IOC) are related to the general operation of an airline. These components comprise advertising, ticketing, sales, reservations, administration, and passenger services. Most studies employ a very crude estimate between total indirect to direct costs of 1:1, which proves to be too simplistic because this ratio is reliant on the type of market in which the airline operates. For example, carriers servicing mature markets generally have higher IOC:DOC compared to low-cost airlines because of much higher passenger services and marketing costs. Thus, it is intended that a more detailed model for these costs should be employed. The information to follow proposes more realistic associations of these additional cost components.

Agent's Commission and Excess Baggage

This incurred cost is dependent on the volume of paying customers; it can be deduced that a cost model in units of CU/SECT proportional to the total revenue would be applicable,

$$C_{\mathrm{pax}} = k_{\mathrm{IOC}}^{\mathrm{I}} Y_{\mathrm{SEC}} \tag{27}$$

where $k_{\mathrm{IOC}}^{\mathrm{I}}$ is a constant representing some factor of Y_{SEC} per sector (CU/SECT).

Sales and Reservation

When a coupling to the total number of revenue passenger-miles, is assumed, this indirect cost contribution (CU/SECT) due to sales and reservation becomes

$$C_{\mathrm{sales}} = k_{\mathrm{IOC}}^{\mathrm{III}} \lambda \,\mathrm{PAX} s \tag{28}$$

where $k_{\mathrm{IOC}}^{\mathrm{III}}$ is a cost function factor in relation to the quantity of available seat-miles (CU/ASM).

Miscellaneous Indirect

This entry can be considered a contingency cost (advertising, passenger handling, administration, etc.) not covered by any other formal definitions. The miscellaneous indirect also can be regarded as a reflection of an airliner's efficiency. Because, as elucidated before, there are occasions where it is desirable to consider the indirect proportional to the direct cost, the supposition is the effectiveness of a carrier with respect to this cost constituent can be represented by a compound function of sector distance (or a direct function of sector DOC) and a fixed component. When $k_{\mathrm{IOC}}^{\mathrm{II}}$ is defined as a constant depicting the fraction of miscellaneous indirect to the DOC for a given sector mission, and $k_{\mathrm{IOC}}^{\mathrm{IV}}$ is defined as an incremental contribution independent of all sources,

$$C_{\mathrm{misc}} = k_{\mathrm{IOC}}^{\mathrm{II}} C_{\mathrm{DOCS}} + k_{\mathrm{IOC}}^{\mathrm{IV}} \tag{29}$$

the miscellaneous IOC component has units of CU/SECT.

TOC for a Given Sector Mission

The TOC per sector is the sum of the DOC and IOC. On substitution of all constituents and grouping, the result is

$$C_{\mathrm{TOCS}} = k_{\mathrm{IOC}}^{\mathrm{I}} Y_{\mathrm{SEC}} + \left(1 + k_{\mathrm{IOC}}^{\mathrm{II}}\right) C_{\mathrm{DOCS}} + k_{\mathrm{IOC}}^{\mathrm{III}} \lambda \,\mathrm{PAX} s + k_{\mathrm{IOC}}^{\mathrm{IV}} \tag{30}$$

P-ROI

The profit for a given sector mission and flight technique is given by the difference between revenue and TOC,

$$P_{\mathrm{SEC}} = Y_{\mathrm{SEC}} - C_{\mathrm{TOCS}} \tag{31}$$

When a generalized total utilization of N_s sectors per given reference time frame is assumed, total profit P is

$$P = N_s\left(1 - k_{\mathrm{IOC}}^{\mathrm{I}}\right) Y_{\mathrm{SEC}} - N_s\left[\left(1 + k_{\mathrm{IOC}}^{\mathrm{II}}\right) C_{\mathrm{DOCS}} + k_{\mathrm{IOC}}^{\mathrm{III}} \lambda \,\mathrm{PAX} s + k_{\mathrm{IOC}}^{\mathrm{IV}}\right] \tag{32}$$

where N_s is substituted by Eqs. (1) or (2) if an hourly based utilization is assumed. Otherwise, N_s becomes a presupposed fixed number of sectors for the arbitrary reference time frame considered. It is expected that the identification of profit optimal flight techniques will not depend on the total ownership and crew salary. This is evident because the profit calculated applies for the entirety of the reference time frame in question; therefore, it is not anticipated to impart any influence because it has become decoupled from aircraft productivity.

It is sound practice for any airline to gauge the relative economic feasibility of potential equipment types by comparing a ratio calculated as the difference between revenue and TOC normalized by the initial investment cost in vehicle acquisition. The ROI (P') can be algebraically expressed as

$$P' = P/c_{\text{acint}} \tag{33}$$

Because the profit result is simply normalized by c_{acint} (here not equivalent to the depreciation cost) to derive ROI, the profit optimal flight technique algorithm to follow is equally applicable for identification of maximum ROI as well. Note that even though conditions for P-ROI optimality are identical, divergent conclusions about the feasibility of an equipment type against another might arise, that is, by virtue of comparisons using absolute currency units vs a nondimensional result.

Flight Technique Optimization for Given Mission

It is evident that any identified optimal flight technique will fall into one of two distinct categories: applicability for an hourly based reference time frame, or fixed-departures-based utilization. It would be of interest to see if the qualitative conjectures drawn earlier about the differences between these two utilization premises will eventuate after analytical scrutiny.

Cost Optimal Flight Technique Identification

It has been shown that the total DOC is basically a function of block time. Thus, accomplishing the task of identifying an optimal cost flight technique depends primarily on solving for a block time that yields minimum cost. An optimum condition is defined by the criterion

$$\frac{\mathrm{d}C_{\text{DOCS}}}{\mathrm{d}t} = 0 \tag{34}$$

Hourly Based Reference Time Frame Utilization

For minimum cost, a block time is selected that minimizes C_{DOCS}. When the derivative of the total DOC per sector mission is set equal to zero, on manipulation, an interim result becomes

$$p_f W_{f,\text{mintime}}(1 - k_1)k_2 \operatorname{sech}^2[k_2(t_{\text{mintime}} - t)] = \varpi \tag{35}$$

where ϖ consists of the remaining variables on the right-hand side of the differential of Eq. (25), namely,

$$\begin{aligned}\varpi|_{\text{DOC}} = {} & c_{\text{conb}} + c_{\text{conf}} + \dot{c}^c_{a\,\text{main}} + \dot{c}^c_{p\,\text{main}} + \dot{c}^c_{a\,\text{mat}} + \dot{c}^c_{p\,\text{mat}} \\ & + p_f W_{f,\text{minfuel}}(1 - k_3)k_4 \operatorname{sech}^2[k_4(t_{\text{minfuel}} - t)] \\ & + \{[c_{\text{lease}}] + c_{\text{acint}} - c_{\text{acres}} + c_{\text{hins}} + c_{\text{crew}} + c_{\text{spint}} - c_{\text{spres}}\}/T_u\end{aligned} \tag{36}$$

The total maintenance cost contribution is given by examining the rate of change of Eq. (15) with respect to block time

$$\dot{c}^c_{\text{main}} = \frac{\mathrm{d}C_{\text{main}}}{\mathrm{d}t} = \left(c_{\text{main}} - \frac{(\beta_{\text{main}} - 1)\alpha_{\text{main}}}{(t - t_{\text{man}})^{\beta_{\text{main}}}}\right)(1 + k_{\text{main}}) \tag{37}$$

This expression can be considered generic and, hence, applicable for both airframe- and propulsion-related cost modeling. It also includes a scope to partition the materiel cost in a similar fashion.

Thus, the block time required for a minimum cost flight operation is

$$t_{\text{mincost}} = t_{\text{mintime}} + \frac{1}{k_2}\cosh^{-1}\sqrt{\frac{p_f W_{f,\text{mintime}}(1 - k_1)k_2}{\varpi|_{\text{DOC}}}} \tag{38}$$

The optimal cost block time is given by a transcendental equation and can be solved numerically via simple iteration. Provided that Eq. (38) passes the Hie latency test (discussed later), the hyperbolic function always aids in achieving quick convergence, and the iterative scheme is inherently stable.

Fixed-Departures-Based Utilization

Because N_s is considered to be a fixed quantity here, it was observed that the total ownership and crew salary would be uncoupled from flight technique and, hence, block time. Under the pretext of Eq. (25), the parameter ϖ in Eq. (35) becomes

$$\begin{aligned}\varpi|_{\text{DOC}} = {} & c_{\text{conb}} + c_{\text{conf}} + \dot{c}^c_{a\,\text{main}} + \dot{c}^c_{p\,\text{main}} + \dot{c}^c_{a\,\text{mat}} + \dot{c}^c_{p\,\text{mat}} \\ & + p_f W_{f,\text{minfuel}}(1 - k_3)k_4 \operatorname{sech}^2[k_4(t_{\text{minfuel}} - t)]\end{aligned} \tag{39}$$

with the corresponding optimal block time found after substitution into Eq. (38).

Derivation of Cost Index

On inspection, it can be readily seen that Eq. (25) may be expressed in the form

$$C_{\text{DOCS}} = C^{\text{I}}_{\text{DOCS}} + p_f W_{\text{fuel}} + C^{\text{III}}_{\text{DOCS}} \tag{40}$$

where each component is the time-related fuel and fixed costs, respectively. Differentiation of C_{DOCS} with respect to block time and on application of the condition for optimality, namely, Eq. (34), gives

$$\frac{\mathrm{d}C_{\text{DOCS}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}C^{\text{I}}_{\text{DOCS}} + p_f\frac{\mathrm{d}W_{\text{fuel}}}{\mathrm{d}t} = 0 \tag{41}$$

Or, conversely, the condition for a cost optimal flight technique occurs when

$$\left.\frac{\mathrm{d}W_{\text{fuel}}}{\mathrm{d}t}\right|_{\text{mincost}} = -\frac{\dot{C}^{\text{I}}_{\text{DOCS}}}{p_f} \tag{42}$$

where, for an hourly based utilization,

$$\begin{aligned}\dot{C}^{\text{I}}_{\text{DOCS}} = {} & c_{\text{conb}} + c_{\text{conf}} + \dot{c}^c_{a\,\text{main}} + \dot{c}^c_{p\,\text{main}} + \dot{c}^c_{a\,\text{mat}} + \dot{c}^c_{p\,\text{mat}} \\ & + \{[c_{\text{lease}}] + c_{\text{acint}} - c_{\text{acres}} + c_{\text{hins}} + c_{\text{crew}} + c_{\text{spint}} - c_{\text{spres}}\}/T_u\end{aligned} \tag{43}$$

otherwise, for fixed departures, $\dot{C}^{\text{I}}_{\text{DOCS}}$ becomes

$$\dot{C}^{\text{I}}_{\text{DOCS}} = c_{\text{conb}} + c_{\text{conf}} + \dot{c}^c_{a\,\text{main}} + \dot{c}^c_{p\,\text{main}} + \dot{c}^c_{a\,\text{mat}} + \dot{c}^c_{p\,\text{mat}} \tag{44}$$

When a cost index (CI) is defined as the rate of change of block fuel per unit block time,

$$\text{CI} = \left|-\frac{\dot{C}^{\text{I}}_{\text{DOCS}}}{p_f}\right| \tag{45}$$

It now becomes possible to examine the relative merits of a given procedural flight technique to the theoretical optimum. In fact, CI describes a gradient magnitude (unit block fuel per unit block time) coinciding with the point where minimum cost occurs on a block time–fuel summary and, advantageously, is independent of sector distance, the mission characteristic of payload, and ambient conditions. This parameter shows consistency with the CI definition for fixed departures utilization found elsewhere in literature.[8,9]

P-ROI Optimal Flight Technique Identification

For maximum P-ROI, a flight technique is selected that maximizes profit P (or P'). For a given sector mission, this condition is governed by the following criterion

$$\left(\frac{\partial P}{\partial N_s}\right)_s = \left(\frac{\partial P}{\partial t}\right)_s\left(\frac{\partial t}{\partial N_s}\right)_s \tag{46}$$

Equation (46) holds true for instances where the reference time frame total utilization is expressed in operating or BH. It can be surmised, because the number of sectors per given reference time frame is dependent on flight technique, that the variable t, or block time, shall impart a corresponding rate change in P, steadily increasing until a maximum stationary point is reached. This represents a partially constrained optimum because the P-ROI expression actually imposes dual criteria that is not only flight-technique dependent for given sector mission, but, as a consequence, is a function of the number of available seat-miles, thus implying the existence of a global optimum at an appropriate sector distance. It is envisaged that the condition for optimal P-ROI will produce a lower block time requirement compared to its cost optimal counterpart, or faster flight techniques that tend more toward the minimum time threshold.

Alternatively, if a fixed number of departures for the given reference time frame is considered, the quantity N_s is no longer coupled to flight technique, hence defining maximum P-ROI via the condition where minimum DOC occurs. This can be substantiated algebraically through manipulation of Eq. (46) into the form shown:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \left(\frac{\mathrm{d}P}{\mathrm{d}N_s}\right)\left(\frac{\mathrm{d}N_s}{\mathrm{d}t}\right) \equiv \frac{\mathrm{d}C_{\mathrm{DOCS}}}{\mathrm{d}t} = 0 \tag{47}$$

Because a measure of P-ROI becomes dependent on the level of DOC for given block time, maxima identification results in a condition where maximum P-ROI is synonymous with minimum DOC.

Hourly Based Reference Time Frame Utilization

When the partially constrained optimum condition given by Eq. (46) is applied to the definition for P-ROI in Eq. (32),

$$p_f\, W_{f,\mathrm{mintime}}(1-k_1)k_2\,\mathrm{sech}^2[k_2(t_{\mathrm{mintime}}-t)] = \varpi \tag{48}$$

where ϖ consists of the remaining variables on the right-hand side of the differential of Eq. (32).

Further scrutiny of ϖ allows for one possible simplification to be in the form of a quotient

$$\varpi|_{\text{P-ROI}} = \varpi' \Big/ \left(1 + k^{\mathrm{II}}_{\mathrm{TOC}}\right)(t + t_a) \tag{49}$$

with the variable located in the numerator defined as

$$\begin{aligned}\varpi' = {} & \left(1 - k^{\mathrm{I}}_{\mathrm{IOC}}\right) y_1 \lambda\,\mathrm{PAX} s\{1 + y_2 \tanh[y_3(s_{\mathrm{ref}} - s)]\} \\ & - \Big(\left(1 + k^{\mathrm{II}}_{\mathrm{IOC}}\right)\Big\{c_{\mathrm{pins}}\lambda\,\mathrm{PAX}s - c_{\mathrm{conf}}\, t_{\mathrm{man}} - (c_{\mathrm{conb}} + c_{\mathrm{conf}})t_a \\ & + \dot{c}^{p}_{a\,\mathrm{main}} + \dot{c}^{p}_{p\,\mathrm{main}} + \dot{c}^{p}_{a\,\mathrm{mat}} + \dot{c}^{p}_{p\,\mathrm{mat}} + p_f W_{\mathrm{fuel}} \\ & - p_f W_{f,\mathrm{minfuel}}(1-k_3)k_4(t+t_a)\,\mathrm{sech}^2[k_4(t_{\mathrm{minfuel}} - t)] \\ & + c_{\mathrm{land}} + c_{\mathrm{nav}} + c_{\mathrm{grh}} + c_{\mathrm{misc}}\Big\} + k^{\mathrm{III}}_{\mathrm{IOC}}\lambda\,\mathrm{PAX}s + k^{\mathrm{IV}}_{\mathrm{IOC}}\Big)\end{aligned} \tag{50}$$

One important piece of information gleaned on perusal of Eq. (50) is that aircraft ownership and crew salary have now become uncoupled from the P-ROI optima identification process. A unique nomenclature to represent the rate change of maintenance cost, with respect to the number of sectors completed for a given hourly based reference time frame utilization, was derived to be

$$\begin{aligned}\dot{c}^{p}_{\mathrm{main}} = \frac{\partial C_{\mathrm{main}}}{\partial N_s} = {} & -c_{\mathrm{main}}(t_{\mathrm{man}} + t_a) \\ & + \frac{\alpha_{\mathrm{main}}}{(t - t_{\mathrm{man}})^{\beta_{\mathrm{main}}}}[\beta_{\mathrm{main}}(t + t_a) - (t_{\mathrm{man}} + t_a)]\end{aligned} \tag{51}$$

This expression can be considered generic. Hence, it is applicable for both airframe- and propulsion-related cost modeling and also includes a scope to partition the materiel cost in a similar fashion.

Now, the block time required for a maximum P-ROI operation is given by

$$\text{P-ROI} = t_{\mathrm{mintime}} + \frac{1}{k_2}\cosh^{-1}\sqrt{\frac{p_f W_{f,\mathrm{mintime}}(1-k_1)k_2}{\varpi|_{\mathrm{ROI}}}} \tag{52}$$

As for optimal cost, the maximum P-ROI block time is given by a transcendental equation of similar form, but additionally influenced by revenue, an IOC component, and turn-around time contributors within the ϖ transient. Once again, this can be solved numerically via simple iteration. Equation (52) must also adhere to rules governed by the Hie latency test (discussed later).

Fixed-Departures-Based Reference Time Frame Utilization

As was shown, Eq. (46) gives the partially constrained optimal block time for hourly based utilization. Based on this premise, it was shown thereafter via Eq. (47) that a maximum P-ROI flight technique for fixed-departures-based reference time frame utilization would be equivalent to a cost optimal procedure, thus, the solution is given by Eq. (52), but with a revised definition of ϖ in Eq. (49):

$$\begin{aligned}\varpi|_{\text{P-ROI}} \equiv \varpi|_{\mathrm{DOC}} = {} & c_{\mathrm{conb}} + c_{\mathrm{conf}} + \dot{c}^{c}_{a\,\mathrm{main}} + \dot{c}^{c}_{p\,\mathrm{main}} + \dot{c}^{c}_{a\,\mathrm{mat}} + \dot{c}^{c}_{p\,\mathrm{mat}} \\ & + p_f W_{f,\mathrm{minfuel}}(1-k_3)k_4\,\mathrm{sech}^2[k_4(t_{\mathrm{minfuel}} - t)]\end{aligned} \tag{53}$$

Hie Latency Index

Because of the form of Eqs. (38) and (52), it can be deduced that a limitation of the inverse hyperbolic cosine function occurs for cases where the variables within the functions collectively produce numbers less than unity. This condition is analogous to a situation where an unconstrained DOC minimum or P-ROI maximum simply does not exist, thus implying that only the quickest flight technique (minimum time) is applicable.

The concept of a Hie latency test (HLT) is presented here as a hypothesis-based testing procedure to help identify the aforementioned circumstance. For DOC and P-ROI optima, regardless of reference time frames, the Hie latency index (HLI) is defined as

$$\mathrm{HLI} = p_f W_{f,\mathrm{min\,time}}(1-k_1)k_2/\varpi \tag{54}$$

where ϖ conforms to definitions based on the type of reference time frame utilization and cost–profit modeling premise. The HLT is then governed by the following criteria:

1) When HLI > 1, a unique solution other than the minimum time flight technique exists (unconstrained optima).

2) When HLI ≤ 1, only the minimum time flight technique is applicable (constrained optima).

It is emphasised that the HLT must be conducted while assuming minimum time flight technique block times.

For HLIs less than or equal to unity, the absence of an unconstrained DOC or P-ROI optimal flight technique is viewed as being unfavorable. Such a result implies block speeds faster than the lowest block time threshold physically permissible by the given vehicle is required to attain a true DOC or P-ROI optimum. Additionally, no operational flexibility is afforded when air traffic control (ATC) or route structure impose off-optimal restrictions. Therefore, whenever scrutiny of en route performance is conducted, the objective of any operationally balanced design should be avoidance of such a situation, particularly for short-haul missions where there is a propensity for faster block speeds.

Operational Flexibility Index

The HLT is a useful tool in qualitatively assessing any penchant an aircraft has for flying faster in achieving economically optimal results. However, this parameter does not provide the analyst or designer with a true perspective of a given vehicle's operational flexibility and, as is the case with CI, a computed value of the HLI parameter is not universally comparable between aircraft of varying scale and propulsion philosophy. One suggestion is to inspect the nondimensional ratio of optimal block time against the minimum fuel and minimum time flight technique block time bandwidth. Because Eqs. (38) and (52) are algorithms solving for optimal block time referenced to minimum time, a possibility now arises in the formulation of an operational flexibility index (OFI)

$$\mathrm{OFI} = \frac{\cosh^{-1}\sqrt{\mathrm{HLI}}}{k_2(t_{\mathrm{minfuel}} - t_{\mathrm{mintime}})} \tag{55}$$

It is evident that a limitation arises for HLI values less than unity in Eq. (55) because of the trigonometric properties displayed by hyperbolic cosine functions; cosh x varies from $-\infty$ to $+1$ to $+\infty$, and $\cosh 0 = 1$. Notwithstanding, such an occurrence signifies that the optimal flight technique corresponds to minimum time flight and can, thus, be considered equivalent to OFI $= 0$. To appreciate the extent of operational flexibility contemporary vehicles offer, typical values of OFI for various aircraft, economic objective function, and utilization assumptions are itemized as follows: 1) DOC and P-ROI optimal hourly based utilization, OFI ≤ 0.15 for regional aircraft and OFI $\cong 0.75$ for narrow and wide bodies and 2) DOC and P-ROI optimal fixed departures utilization, OFI $\cong 0.20$ for regional aircraft, and OFI $\cong 0.90$ for narrow and wide bodies.

A design condition OFI value approaching zero denotes little or no scope for flexibility because it is congruous with minimum time flight techniques. Not only does this condition usually deny the possibility of achieving unconstrained optima, but also implicitly dictates that all shorter-range operations will follow suit. Additionally, this circumstance is seen to be detrimental because the criterion of a higher engine rating flight technique may reduce the service life of the powerplant. It does, however, allow for longer-range mission capability without trading payload for fuel, but at an ever-increasing penalty of off-optimality as distance becomes longer.

A maximum value of OFI $= 1.00$ at the design condition, akin to a minimum fuel technique, affords a limited range of operational flexibility on the other end of the spectrum. Even though a scope is given for the generation of unconstrained optima flight techniques for shorter sector distances, useful load limitations may not permit the opportunity of longer-range missions for a given payload. This would necessitate an exchange of payload for increased range, thereby limiting the potential for revenue.

A salient objective would be a design OFI $= 0.50$ for any prospective aircraft evaluation exercise. This will ensure avoidance of premature useful load limitations for longer sector distances and, importantly, increase the likelihood of unconstrained optima for shorter distances. Finally, the penalties associated with off-optimal flight techniques commonly experienced in actual operation can be minimized.

Economical LRC

Traditionally, LRC has been understood to be 99% (sometimes even 98%) of MRC SAR toward the faster end of the curve.[16–19] This practice is employed to trade increased speed capability for what is considered to be a relatively small penalty in fuel consumption rate. Indeed, after the inception of this rule-of-thumb procedure for en route performance analysis, it has now become a mainstay technique in industry circles. It would be of interest to see how this popular assumption measures up against speed technique formulation using economic criteria alone.

Initially, an objective function for what constitutes economical cruise must be formulated. One candidate is to use a fixed departures utilization assumption. Not only is this a consistent basis of emulating actual operator scheduling, but also, as outlined before, this premise theoretically generates optimal flight techniques that are slower than an hourly based utilization. Even though CI represents a necessary magnitude of $\mathrm{d}W_{\mathrm{fuel}}/\mathrm{d}t$ that ensures cost optimality for any sector mission criteria, an approximate expression explicitly related to cruise speed and SAR can also be derived. Assuming cruise fraction is sufficiently large, thus neglecting the influence of climb and descent, it can be demonstrated[16] that

$$\mathrm{CI} = \left|\frac{\mathrm{d}W_{\mathrm{fuel}}}{\mathrm{d}t}\right| \approx \left|a_{\mathrm{sls}}\theta^{\frac{1}{2}}\left(\frac{M}{\mathrm{SAR}}\right)^{2}\frac{\mathrm{d\,SAR}}{\mathrm{d}M}\right| \tag{56}$$

Figure 7 shows the degradation of SAR compared to the MRC datum for regional, narrow-body and wide-body twins using computed CIs of 10, 25, and 40 [such speed techniques are henceforth dubbed economical long-range cruise (ELRC)], respectively. These values were based on a projected fuel price and known operator time-dependent maintenance cost data. Note that a standard representation of CI assumes a value normalized by 100 lb/h (Refs. 8 and 9). On comparison to a 1% reference line denoting the contemporary LRC assumption, it is observable that a large disparity between LRC and ELRC takes place. It is immediately evident that the SAR curve is quite flat for lower flight levels, promoting even larger deviations from the conventional 1% degradation. However, for narrow and wide bodies at typical cruise altitudes in excess of 29,000 ft, where optimal cruise begins and, subsequently, resides in the drag rise region, ELRC dictates speed schedules around 99.5% maximum SAR. Inspection of Eq. (56) lends support to this phenomenon. If one appreciates that $\mathrm{d\,SAR}/\mathrm{d}M$ measurably increases in magnitude for flight within the drag rise at fixed CI and altitude, the resulting speed schedule candidate must be reconciled toward MRC. Regional aircraft appear to reach a constant value of 97% maximum SAR at higher altitudes, and this is attributable to drag rise effects generally not being prevalent.

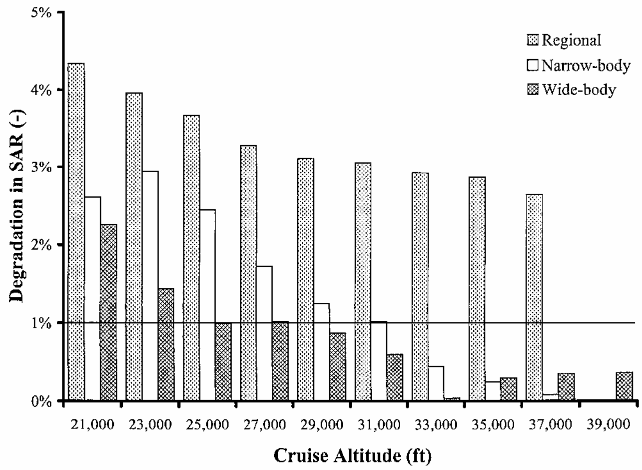

Fig. 7 Degradation in SAR assuming traditional LRC (1% reference line) and ELRC compared to datum of MRC (fixed all-up weight, ISA, and still air).

On perusal of Fig. 7, for both twin narrow-body and wide-body equipment types, adopting the slower ELRC schedule as opposed to LRC amounts to an almost 1% integrated mission flight fuel reduction because the technique is closer to an optimal SAR condition. Correspondingly, the difference between LRC and ELRC equates to a speed reduction of approximately 5 kn true air speed at typical flight plan altitudes. Today, there exists a capacity for operators to soak the slight increase in flight time due to a slower speed, especially now that scheduled block times have been widened to improve on-time dependability and now that fuel prices are on the rise. For occasions where block times must be reduced for the sake of dependability, the ELRC method is congruous with a flight planning system (FPS) increased block speed iteration scheme because the starting point is slightly slower than traditional LRC in any case. In spite of the speed margin to MRC being rationalized on application of an ELRC schedule for narrow and wide bodies, the buffer is still greater than 5 kn CAS. This is a margin commonly assumed for contemporary flight management computer (FMC) en route operational software, and, from an operational perspective, the margin is not deemed prohibitive in terms of speed stability (excursions due to wind shift, turbulence, etc.) in maintaining the target level.

Merit Functions to Measure Relative P-ROI

An interesting feature of the derivation for optimal P-ROI block times assuming hourly based reference time frame utilization is that these solutions are partial optima due to a codependence on block time and quantity of available seat-miles completed by the vehicle. Figure 8 shows typical variation of P-ROI against block time for a variety of sector distances. Important facets of this representation include a distinct P-ROI global optimum and the existence of a break-even sector distance corresponding to an associative block time.

Even though the hourly based reference time frame utilization can be considered idealistic compared to the more pragmatic assumption of a fixed number of sectors, it can provide valuable insight. One important conclusion is that the comparison of different equipment types for only one fixed sector is not a sound enough basis to rationalize the superiority of an aircraft over another. A practical application would be use of this approach as a work tool that aids in maximizing utilization of a given vehicle for existing markets. Another is the possibility of showing the relative merits associated with the introduction of new projected markets involving

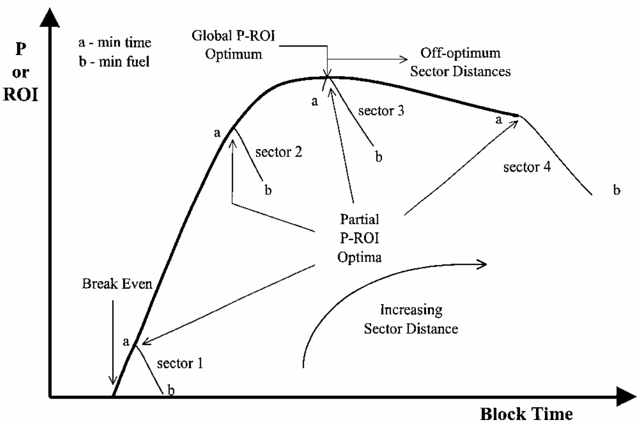

Fig. 8 Typical P-ROI vs block time summary for a variety of sector distances assuming an hourly based reference time frame utilization.

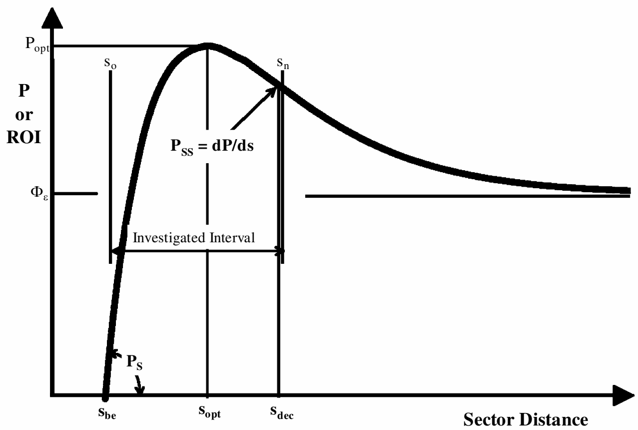

Fig. 9 Typical sector distance response of P-ROI model assuming an hourly based reference time frame utilization.

either variations in sector distance, or mission criteria, or both. A further review potential includes the possibility of conducting detailed competitor studies where economic flexibility can be weighed between the vehicles taken into consideration. The section to follow suggests such guidelines via the introduction of a P-ROI model that can enable association between aircraft performance qualities to the model's geometric attributes and, thus, enable procedural methods of suboptimizing for more desirable P-ROI characteristics.

Method for Comparison of Contemporary Vehicles and Review of Aircraft Designs

A comprehensive method in identifying optimal flight techniques with respect to DOC and P-ROI enables the analyst to quantify the earning potential for given vehicles and mission criteria. Because it has been shown that sector distance can be regarded as an independent variable, it would be of interest to evaluate the various merits of an output P-ROI response with respect to distance. An adequate model for representing the P-ROI coverage in the closed sector distance interval $[s_o, s_n]$ is presented here as

$$P = (\Phi_\alpha s - \Phi_\beta)\exp[-(\Phi_\chi + \Phi_\delta s)] + \Phi_\varepsilon \tag{57}$$

where both P and P' are applicable for the basic structure of Eq. (57).

The equation coefficients represent quantities that enable a possibility of evaluating the properties of a vehicle's flexibility in earning potential through a geometric interpretation. If the model given is actually taken into consideration as an open interval, for example, $[s_{\text{be}}, \infty)$, or, from a break-even sector distance and upward, one can identify uncanny similarities to a typical step response of stable linear control systems (Fig. 9), in this particular instance, the reference input being sector distance and P-ROI the output. It contains a transient response due to sector distance that also includes tendency to an asymptote as sector distance becomes large and exceeds distances constrained by useful load limitations (akin to steady state) and a lower threshold where break-even occurs, or zero P-ROI at some sector distance value.

Typical performance criteria can be formulated that characterize the transient response. These are proposed here as break-even sector distance and corresponding preoptimum P-ROI rise rate, the P-ROI global optimum and corresponding sector distance, measure of the postoptimum P-ROI decay rate, and the magnitude of the model asymptote value. This can be achieved mostly through inspection of the first and second derivatives of Eq. (57).

Break-Even Sector Distance

The break-even sector distance is defined as the sector distance that creates a condition where the P-ROI is zero. Equation (57) is in a form where an exact formula for solving $P(s) = 0$ is not available. The Newton–Raphson method would be a suitable way of approximating such solutions, and this iterative numerical method is projected to give an answer very close to the actual result in a single step with an apt initial estimate such as the lower threshold of the surveyed sector distances, that is, $s_i = s_o$. Using Eq. (57) and the corresponding first derivative in consort with Newton–Raphson produces

$$s_{\text{be}} = s_i - \frac{\Phi_\alpha s_i - \Phi_\beta + \Phi_\varepsilon \exp(\Phi_\chi + \Phi_\delta s_i)}{\Phi_\alpha - \Phi_\delta(\Phi_\alpha s_i - \Phi_\beta)} \tag{58}$$

The break-even sector distance s_{be} should ideally be minimized.

Preoptimum P-ROI Rise Rate

P-ROI increases with greater sector distance for sector distances larger than break-even. The reciprocal of the rate P-ROI increase with respect to sector distance would give a measure of what increment in sector distance achieves a target increase in P-ROI before the global maximum threshold is crossed. An adequate representation of the preoptimum P-ROI rise rate may be given by the instantaneous slope at the break-even sector distance via the definition found in Eq. (58)

$$P_S = \left[\left.\frac{\mathrm{d}P}{\mathrm{d}s}\right|_{s_{\text{be}}}\right]^{-1} = \frac{\exp(\Phi_\chi + \Phi_\delta s_{\text{be}})}{\Phi_\alpha - \Phi_\delta(\Phi_\alpha s_{\text{be}} - \Phi_\beta)} \tag{59}$$

P_S (or P'_S) is a quantity expressed as distance covered per unit P-ROI and should be minimized.

Global P-ROI Optimum Sector Distance

Yet another fundamental observation that can be extracted from Eq. (57) is the identification of sector distances that yield global P-ROI maxima (P_{opt} or P'_{opt}). The distance where a P-ROI global optimum occurs is given by the first derivative of Eq. (57); hence,

$$s_{\text{opt}} = \Phi_\beta/\Phi_\alpha + 1/\Phi_\delta \tag{60}$$

Having s_{opt} as low as possible while simultaneously maximizing P_{opt} (or P'_{opt}) would be the primary goal of any prospective vehicle or design proposal.

Postoptimum P-ROI Decay Rate

Note that P-ROI decreases with increasing sector distance once the P-ROI global optimum sector distance has been surpassed. The rate reduction in P-ROI with respect to sector distance conducted at the inflection point between the postoptimum transient and steady-state responses is proposed here as a useful merit parameter. When the second derivative of Eq. (57) is utilized and sector distance solved for at this inflection point,

$$s_{\text{dec}} = \Phi_\beta/\Phi_\alpha + 2/\Phi_\delta \tag{61}$$

Table 1 Block time–fuel summaries derived for three regional equipment types completing 200–800 n mile sector distances (JAR OPS-1 rules)

Sector, n mile	$W_{f,\mathrm{mintime}}$, kg	k_1	k_2, min^{-1}	t_{mintime}, min	$W_{f,\mathrm{minfuel}}$, kg	k_3	k_4, min^{-1}	t_{minfuel}, min	k_5, kg
Turbofan 1									
200	863	0.912	0.723	40.4	736	1.483	0.0203	47.5	812
350	1360	0.881	1.164	59.4	1057	2.050	0.0128	69.1	1215
500	1866	0.849	0.817	78.2	1376	1.659	0.0160	91.5	1657
800	2727	0.841	0.377	115.9	2033	1.876	0.00680	137.5	2467
Turbofan 2									
200	772	0.928	1.468	43.0	679	1.480	0.0192	49.2	734
350	1244	0.821	0.932	63.4	966	1.472	0.0161	70.8	1188
500	1727	0.788	1.130	83.8	1268	1.835	0.00810	94.7	1633
800	2203	0.874	0.514	125.0	1887	0.825	−0.00740	141.9	2165
Turboprop									
200	705	0.974	1.726	46.8	600	1.909	0.0183	55.5	618
350	1116	0.851	0.397	70.9	874	1.361	0.0165	85.8	1041
500	1531	0.835	0.328	95.1	1155	1.371	0.0136	116.7	1408
800	2362	0.827	0.248	143.5	1724	1.405	0.00981	178.0	2132

Table 2 Cost and yield modeling

Parameter	Yield
Ownership period	10 years
Aircraft and spares inventory interest	10%
Aircraft and spares inventory residual value	40%
Spares ownership	15%
Hull insurance	1%
Total ownership per year	
Turbofan 1	U.S. \$2.87 million
Turbofan 2	U.S. \$2.63 million
Turboprop	U.S. \$2.23 million
Fuel cost	U.S. \$0.60/U.S. gal.
Flight crew (pilots and flight attendants)	\$250 per BH
Annual utilization	Fixed at 2500 operational hours
Turn-around time	30 min

Hence, the postoptimum decay rate (P_{SS}) is given by the slope of the sector distance response

$$P_{\mathrm{SS}} = \left.\frac{\mathrm{d}P}{\mathrm{d}s}\right|_{s_{\mathrm{dec}}} = -\Phi_\alpha \exp\left(-\Phi_\chi - \frac{\Phi_\beta \Phi_\delta}{\Phi_\alpha} - 2\right) \tag{62}$$

P_{SS} (or P'_{SS}) is a quantity that is always negative; therefore, it should be maximized to reduce the potential P-ROI loss rate per unit distance flown.

Worked Example for Regional Equipment

An illustration of the presented methods will be given for two turbofans and a high-speed turboprop of equal maximum accommodation. The analysis to follow is based on aircraft covering sector distances between 200 and 800 n miles, within the European operational environment, employing a Joint Airworthiness Requirements (JAR) OPS-1 (operational procedures for commercial traffic with fixed wings) reserves fuel policy (30 min hold at 1500 ft pressure altitude, 100 n mile diversion, and 5% trip fuel) and a complement of 60% load factor (maximum accommodation has intentionally not been divulged) at 99 kg each per PAX.

En Route Performance

This survey consists of basic block time–fuel summaries derived from batch calculations subsequently stripped of those flight techniques not describing the lower bound of height–energy–block fuel minima for fixed block times. The following parameters derived via nonlinear regression techniques are presented in Table 1.

Various Economic Assumptions

The primary assumptions for cost and yield modeling can be found in Table 2. Sundry expenses that include navigation, landing and handling charges that are fixed for each sector mission have been assumed as shown in Table 3.

Table 3 Synopsis of various sundry expenses incurred for three regional equipment completing 200–800 n mile sector distances

Sector distance, n mile	Sundry cost, U.S. dollars: Turbofan 1	Turbofan 2	Turboprop
200	672	640	671
350	781	743	779
500	890	846	888
800	1108	1052	1104

Maintenance Model

A maintenance model conforming to the time-dependent and cyclic constructs given by Eqs. (10) and (11) in U.S. dollars per flight hour for the Turboprop is approximated as

$$c_{\mathrm{main}} = 90.3 + 168.1 \big/ (t - t_{\mathrm{man}})^{0.827}$$

where an allowance of $t_{\mathrm{man}} = 10$ min is common to all aircraft in this survey. In accordance with vehicular configuration and size, the turboprop maintenance cost model was factored using $k_{\mathrm{main}} = 0.02$ and 0.055 to formulate the Turbofan 2 and 1 constituents costs, respectively.

Yield and Revenue Model

With the basic form given in Eq. (26), the yield and revenue model for this study is

$$Y_{\mathrm{SEC}} = 0.5180\lambda\ \mathrm{PAX}s\{1 + 0.5283\tanh[0.001489(86.88 - s)]\}$$

where λ is the passenger load factor in this analysis (60%), PAX is the maximum vehicular passenger capacity, and s, the sector distance, varied between 200 and 800 n mile.

IOC Models

To complete the basis for ensuing P-ROI calculations, the following ancillary cost models were used to simulate the total operating cost, that is, DOC + IOC: agent's commission and excess baggage, 11% of yield; sales and reservations office, U.S. \$0.004169/revenue passenger n mile; other indirect costs, 14% of DOC.

Synopsis of the Flight Technique Optima

Tables 4–6 give an overview of the cost and profit optimal flight techniques associated with the vehicles investigated in this survey.

A perspective on the operational flexibility of Turbofan 1, 2, and the Turboprop is given in Figs. 10, 11, and 12, respectively. Except for shorter sector distances, that is, less than 350–500 n mile, the

Table 4 Flight technique breakdown for P-ROI global optima assuming an hourly based reference time frame utilization

Vehicle	Sector	CLB[b] mode	CRZ[c] mode	DES[d] mode	Initial CRZ FL[e]	Technique
Turbofan 1	200 n mile	H[f]	MCR[g]	H	250	Min. time[a]
Turbofan 2	200 n mile	CLB	MCR	DES	230	Min. time
Turboprop	200 n mile	H	MCR	H	230	Min. time
Turbofan 1	350 n mile	H	MCR	H	330	Intermediate
Turbofan 2	350 n mile	CLB	MCR	DES	330	Intermediate
Turboprop	350 n mile	H	MCR	H	240	Min. time
Turbofan 1	500 n mile	H	MCR	H	350	Intermediate
Turbofan 2	500 n mile	CLB	MCR	DES	350	Intermediate
Turboprop	500 n mile	H	MCR	H	240	Min. time
Turbofan 1	800 n mile	H	MCR	H	350	Intermediate
Turbofan 2	800 n mile	CLB	MCR	DES	360	Intermediate
Turboprop	800 n mile	H	MCR	H	260	Intermediate

[a]Constrained partial optima. [b]Climb (from two modes). [c] Cruise.
[d]Descent (only one mode). [e]Flight level (100 s ft). [f]High. [g]Maximum cruise.

Table 5 Flight technique breakdown for DOC optima assuming an hourly based reference time frame utilization

Vehicle	Sector	CLB[b] mode	CRZ[c] mode	DES[d] mode	Initial CRZ FL[e]	Technique
Turbofan 1	200 n mile	H[f]	MCR[g]	H	250	Min. time[a]
Turbofan 2	200 n mile	CLB	MCR	DES	250	Intermediate
Turboprop	200 n mile	H	MCR	H	230	Min. time
Turbofan 1	350 n mile	H	MCR	H	330	Intermediate
Turbofan 2	350 n mile	CLB	MCR	DES	330	Intermediate
Turboprop	350 n mile	H	MCR	H	240	Min. time
Turbofan 1	500 n mile	H	MCR	H	350	Intermediate
Turbofan 2	500 n mile	CLB	MCR	DES	350	Intermediate
Turboprop	500 n mile	H	MCR	H	270	Intermediate
Turbofan 1	800 n mile	H	MCR	H	370	Intermediate
Turbofan 2	800 n mile	CLB	MCR	DES	370	Intermediate
Turboprop	800 n mile	H	MCR	H	260	Intermediate

[a]Constrained partial optima. [b]Climb (from two modes). [c] Cruise.
[d]Descent (only one mode). [e]Flight level (100 s ft). [f]High. [g]Maximum cruise.

Table 6 Flight technique breakdown for DOC and ROI optima assuming fixed departures based reference time frame utilization

Vehicle	Sector	CLB[a] mode	CRZ[b] mode	DES[c] mode	Initial CRZ FL[d]	Technique
Turbofan 1	200 n mile	L[e]	INTER[f]	H[g]	280	Intermediate
Turbofan 2	200 n mile	CLB	INTER	DES	250	Intermediate
Turboprop	200 n mile	M[h]	MCR	H	250	Intermediate
Turbofan 1	350 n mile	H	MCR	H	350	Intermediate
Turbofan 2	350 n mile	CLB	MCR	DES	360	Intermediate
Turboprop	350 n mile	M	INTER	H	310	Intermediate
Turbofan 1	500 n mile	H	MCR	H	370	Intermediate
Turbofan 2	500 n mile	CLB	MCR	DES	360	Intermediate
Turboprop	500 n mile	M	INTER	H	310	Intermediate
Turbofan 1	800 n mile	L	INTER	H	370	Intermediate
Turbofan 2	800 n mile	CLB	INTER	DES	370	Intermediate
Turboprop	800 n mile	M	INTER	H	310	Intermediate

[a]Climb (from two modes). [b]Cruise. [c] Descent (only one mode).
[d]Flight level (100 s ft). [e]Low. [f]Intermediate cruise speed. [g]High. [h]Medium

OFI of each vehicle resides between 0.05 and 0.10 for minimum DOC and maximum P-ROI, assuming an hourly based utilization; the fixed-based departures premise elevates OFI values to around 0.20. With respect to an ideal of OFI = 0.50, both of these sets of values are considered to possess unfavorable tendencies toward the faster block speed procedure, limiting opportunities of minimizing the penalty incurred when operating at slower off-optimal flight techniques.

As an exemplar of the flight technique results, Fig. 13 supplies a graphical interpretation using computed optimal block times for given objective on each of Turbofan 1, 2, and Turboprop characteristic block time–fuel curves assuming a 500-n mile sector mission. Within Fig. 13, the minimum fuel point is characterized by a constrained maximum SAR flight technique, that is, the slowest forward speed climb mode, LRC (instead of MRC) en route speed, optimum altitude profile, and the slowest forward speed for descent mode. Correspondingly, the minimum time node is congruous with the fastest block speed achievable for given sector distance, that is, where no fuel limitation is imposed, the flight technique consists of the fastest forward speed climb mode, MCR en route speed, an altitude profile generating the fastest ground speed, and the fastest forward speed for descent mode. The appeal of turbofan aircraft is quite apparent when comparing block times for minimum time and minimum fuel between Turbofan 1, 2, and the Turboprop. As an indication of the speed difference, it is discernable in Fig. 13 that a minimum time flight technique for the Turboprop is equivalent in time to the Turbofan 2 minimum fuel technique.

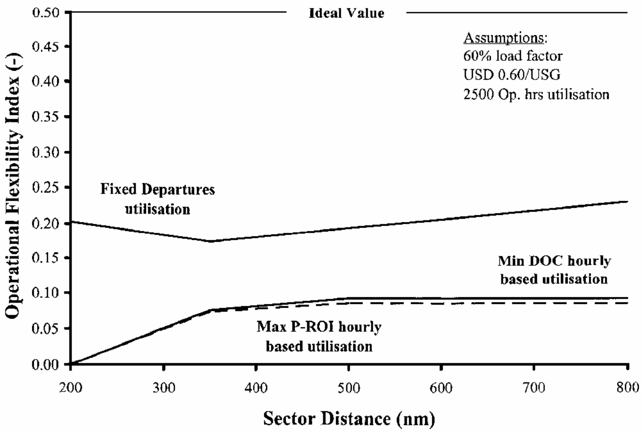

Fig. 10 OFI values corresponding to optimal flight techniques assuming an hourly based, as well as fixed-departures-based, utilization for Turbofan 1 vehicle.

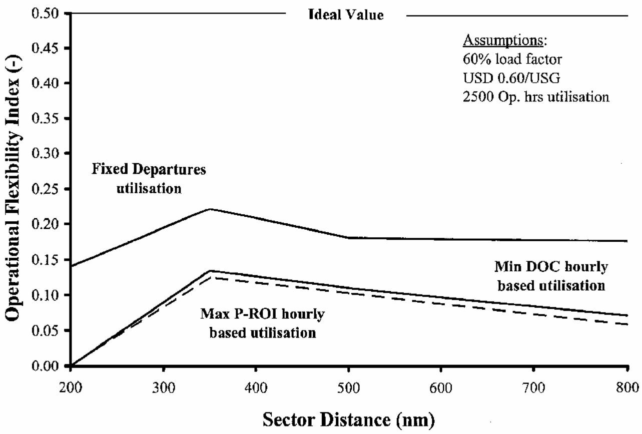

Fig. 11 OFI values corresponding to optimal flight techniques assuming an hourly based, as well as fixed-departures-based, utilization for Turbofan 2 vehicle.

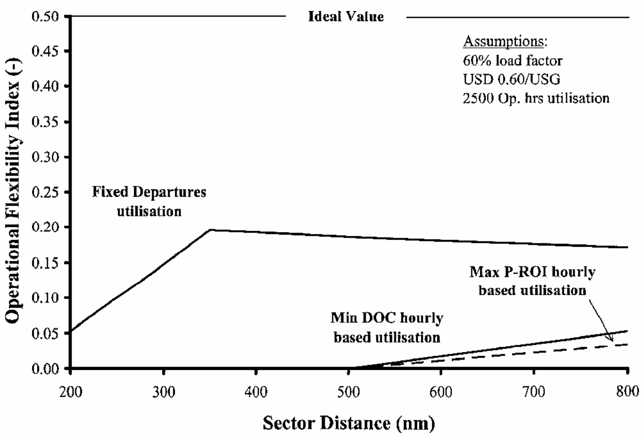

Fig. 12 OFI values corresponding to optimal flight techniques assuming an hourly based, as well as fixed-departures-based, utilization for Turboprop vehicle.

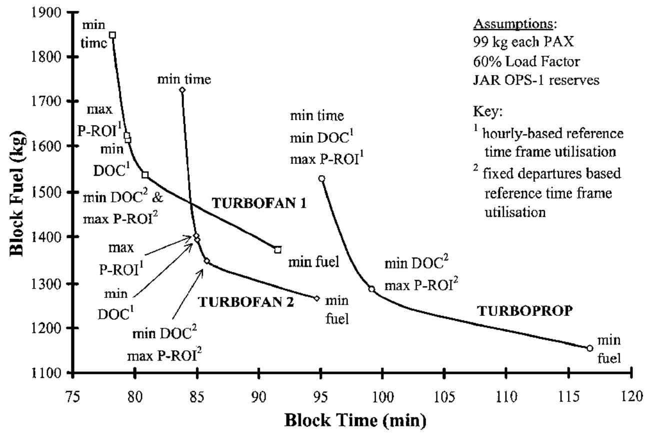

Fig. 13 Block time–fuel summary for three regional equipment types completing 500-n mile sector distances.

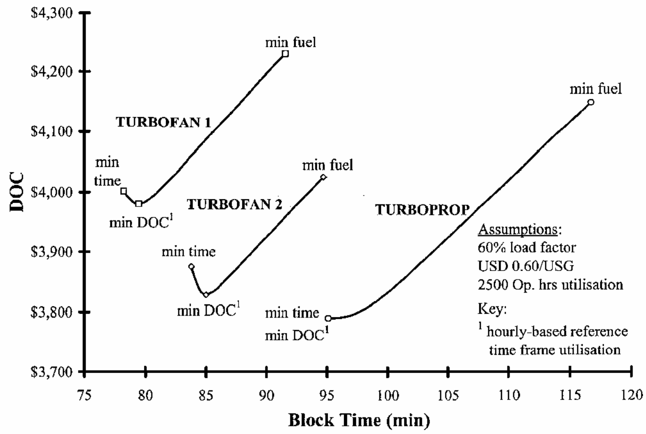

Fig. 14 DOC variation with flight technique for three regional equipment types completing 500-n mile sector distances.

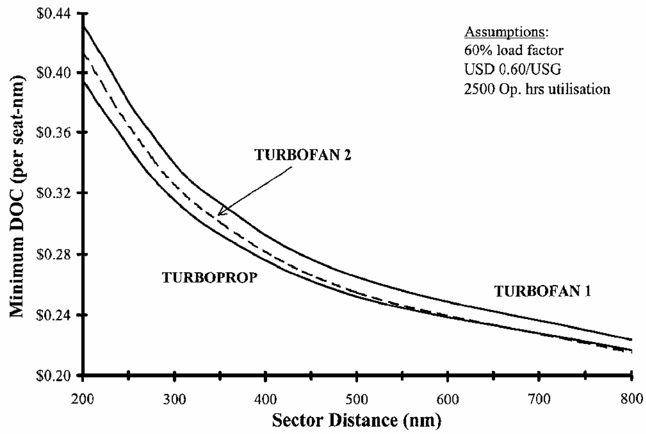

Fig. 15 Minimum DOC against sector distance for three regional equipment types.

Competitive Analysis Between Regional Equipment

Figure 14 shows the relative DOC variation with block time and minimum DOC for Turbofan 1, 2, and Turboprop vehicles assuming a sector distance of 500 n mile and an hourly based reference time frame utilization. It is evident that most of the cost optimal flight techniques are indicative of the unconstrained optima condition with the exception of the Turboprop, wherein a constrained optimum applies, that is, HLI ≤ 1, or minimum-time flight technique. The turboprop exhibits superiority in terms of minimum DOC achievable compared to Turbofan 2, which can be regarded as the next closest rival, as well as over Turbofan 1.

To gauge how well the Turboprop performs against Turbofan 1 and 2 in terms of DOC for a variety of sector missions, Fig. 15 presents computed minimum DOCs per seat-nautical mile for all three forms of regional equipment. The Turboprop maintains a cost-effective posture up to a sector distance of approximately 650 n mile, at which point Turbofan 2 exhibits a marginal advantage.

Given an hourly based reference time frame utilization, Fig. 16 shows the variation of annual profit with block time and identifies the partial profit optima for a sector distance of 500 n mile. The profit optimal flight techniques are indicative of somewhat slightly lower

Table 7 Tabulation of P-ROI regression coefficients and merit parameters for three regional equipment types

Objective		Parameter	Turbofan 1	Turbofan 2	Turboprop
P	Φ_α	U.S. \$ million/n mile	0.01012	0.01013	0.01061
P	Φ_β	U.S. \$ million	2.854	2.825	2.889
P	Φ_χ	——	−0.7061	−0.6520	−0.9862
P	Φ_δ	per n mile	0.003896	0.003899	0.005019
P	Φ_ε	U.S. \$ million	−0.2068	0.2349	0.1104
P-ROI	s_{be}	n mile	317	275	258
P	P_s	n mile/U.S. \$ million	194	148	120
P-ROI	s_{opt}	n mile	539	535	472
P	P_{opt}	U.S. \$ million	0.439	0.642	0.641
P	P_{ss}	U.S. \$/n mile	−925	−888	−981
ROI	Φ'_α	per n mile	0.003522	0.003858	0.004788
ROI	Φ'_β	——	0.9932	1.076	1.304
ROI	Φ'_ε	——	−0.07197	0.008942	0.04982
ROI	P'_s	n mile	556	390	266
ROI	P'_{opt}	——	0.153	0.245	0.290
ROI	P'_{ss}	per 100 n mile	−0.0322	−0.0338	−0.0443

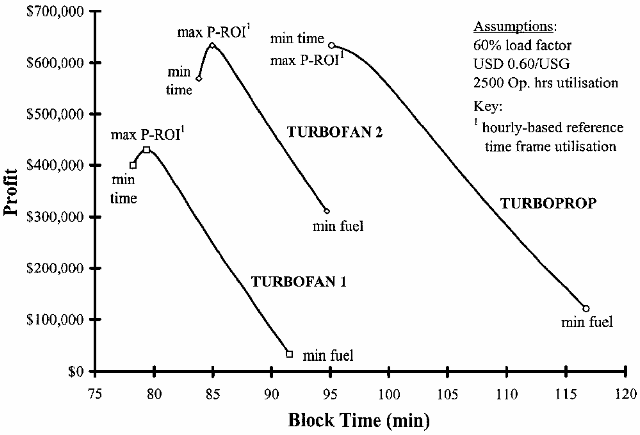

Fig. 16 Profit per annum variation with flight technique for three regional equipment types completing 500-n mile sector distances.

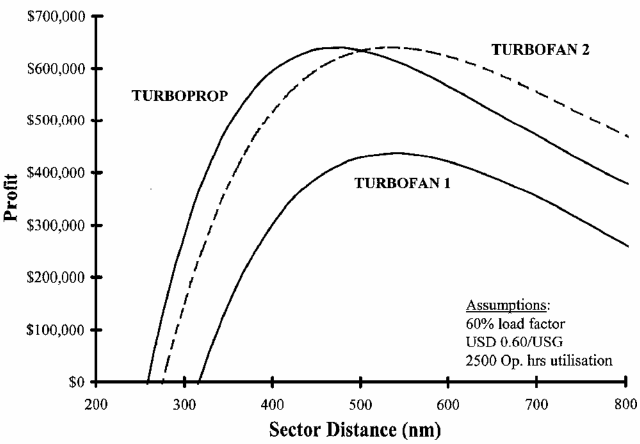

Fig. 17 Maximum profit per annum against sector distance for three regional equipment types.

block times (faster block speeds) compared to their cost optimal counterparts. Turbofan 1 and 2 are characterised by partially unconstrained optima, unlike the turboprop, where a partially constrained optimal flight technique is dictated.

Figure 17 gives the annual potential for profit between the Turbofan 1, 2, and Turboprop vehicles for sector distances up to 800 n mile. Visual inspection of this diagram qualitatively shows the Turbofan 2 slender margin of superiority over the Turboprop with respect to the global maximum profit value.

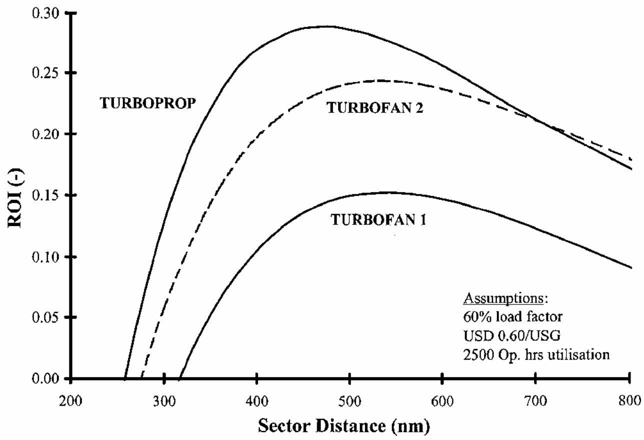

Fig. 18 ROI against sector distance for three regional equipment types.

Alternatively, by employing the analytical model given in Eq. (57), the parameters listed in Table 7 were derived for the vehicles investigated.

On review of the primary merit quantities shown in Table 7, note that Turbofan 2 has the highest and best value of P_{SS} or postoptimum P-ROI degradation characteristics, even though the Turboprop generates higher profit for sector distances less than around 500 n mile. Advantages of the Turboprop vehicle over its counterparts are a P_s or profit rise rate of 120 n mile per U.S. \$ million and s_{be}, or a break-even sector distance value of 258 n mile. Also, the Turboprop appears to have the most desirable P-ROI global optimum sector distance, as exemplified by comparison of s_{opt} values for Turbofan 1, 2, and Turboprop of 539, 535, and 472 n mile, respectively. Notwithstanding the positive attributes of the Turboprop vehicle's optimal P-ROI stage length, the Turbofan 2 value of P_{opt} being the largest, and Φ_ε at least twice as large the closest competitor, combined with aforementioned qualities, signifies this vehicle's superior nature in terms of potential for generating profit and concludes the review as being the better acquisition.

Note that if one ignores the absolute value of profit, but instead examines ROI in isolation (Table 7 and Fig. 18), it is evident that the Turboprop demonstrates superior attributes. This circumstance can be explained by the relatively inexpensive acquisition price of turboprop vehicles in the contemporary market, thus generating a proportionately higher return on investment. Even though this aspect might be construed as a lucrative outcome, the analyst or designer must recognize the significant tradeoff in block speed for sector distances greater than 350 n mile. This fundamental characteristic, therefore, ratifies the widely held notion that turboprops are well suited for shorter sector distance missions only.

Conclusions

This paper outlines a systematic methodology to identify given sector distance mission flight techniques, or an operational protocol consisting of a specific climb speed schedule, initial cruise altitude, cruise speed schedule, step-cruise profile, and descent speed schedule that produce minimum DOC and maximum P-ROI. All cost and yield relationships can be manipulated to suit most contemporary calculation procedures, providing the scope of incorporating a specialized routine into conceptual design mission analysis software.

Some pertinent conclusions drawn from this study concern the relationship of cost and profit optimal flight techniques to one another. An hourly based reference time frame utilization results in distinct flight technique optima for minimum DOC and maximum P-ROI. The P-ROI optima are characterised by faster block speeds than cost optimal ones because of a codependence on flight time and the quantity of available seat-miles completed by the vehicle. Also, the hourly based utilization resulted in partial P-ROI optima for different sector distances, which implied the existence of a global optimum at some specific sector distance and block time. This illustrates that a comparison of distinct equipment types for only one fixed sector is not a sound enough basis to rationalize the superiority of one aircraft over another. A fixed number of sectors' utilization assumption reduces the sensitivity of time-related costs to flight technique and, thus, reduces the significance of this component compared to the fuel expended. This situation produces block speed optima appreciably slower than those assuming an hourly based utilization. Furthermore, the fixed departures assumption creates a condition where both cost optimal and profit optimal flight techniques coincide with one another.

A new speed schedule definition called ELRC was created to replace the traditional 99% maximum SAR LRC speed. The motivation was that not only is the 99% maximum SAR premise inconsistent with cost and profit optimality, but an alternative of simply assuming some other fixed degradation in SAR does not suffice either. It was found that CI is the most suitable method in defining ELRC for the entire gamut of transport aircraft categories available today. To complement this, a merit function called OFI was derived to enable transparency of what en route operational qualities a given aircraft exhibits.

Merit parameters that give rise to the ability of suboptimizing for more desirable P-ROI characteristics were also formulated. Break-even sector distance s_{be} and corresponding preoptimum P-ROI rise rate P_S, the P-ROI global optimum P_{opt} and corresponding sector distance s_{opt}, the postoptimum P-ROI decay rate P_{SS}, together with the magnitude of the asymptote value Φ_ε were suggested as a logical set of guidelines when exploring new conceptual aircraft designs or conducting detailed competitor reviews.

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